

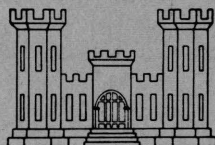
CORPS OF ENGINEERS, U.S. ARMY

HEAT TRANSFER AT THE AIR-GROUND INTERFACE WITH SPECIAL REFERENCE TO AIRFIELD PAVEMENTS

by

Massachusetts Institute of Technology
Department of Civil and Sanitary Engineering
Soil Engineering Division

Contract No. DA-19-016-ENG-2743



TECHNICAL REPORT NO. 63

Under Contract With

Arctic Construction and Frost Effects Laboratory
U.S. Army Engineer Division, New England
Waltham, Massachusetts

for

Office of the Chief of Engineers
Civil Engineering Branch
Engineering Division
Military Construction

January 1961

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CIVIL ENGINEERING BRANCH
ENGINEERING DIVISION
MILITARY CONSTRUCTION

JANUARY 1961

PREFACE

This report is one of three reports describing results of investigations for the 'Design, Fabrication, Exploratory Operation, and Operational Manual of a Hydraulic Analog Computer.' The investigation has been sponsored by the Arctic Construction and Frost Effects Laboratory, U.S. Army Engineer Division, New England, Waltham, Massachusetts, under Contract No. DA-19-016-ENG-2743 with the Massachusetts Institute of Technology through its Division of Industrial Cooperation and DIC Project No. 5-7155.

The entire investigation is directed toward improvement of techniques for predicting subsurface temperatures, particularly with reference to freeze and thaw penetration below airfield pavements. This report describes a study of those variables which affect the transfer of heat at the air-earth interface. Two other reports are (a) Technical Report No. 62, 'Design and Operation of a Hydraulic Analog Computer for Studies of Freezing and Thawing of Soils,' May 1956; and (b) Technical Report No. 67, 'Frost Penetration in Multilayer Soil Profiles,' June 1957.

Contract investigations were conducted under the supervision of Dr. Harl P. Aldrich, Jr., Assistant Professor of Soil Mechanics, Department of Civil and Sanitary Engineering, Massachusetts Institute of Technology. During the first year, Dr. Henry M. Paynter, then Assistant Professor of Hydraulic Engineering, also supervised parts of the investigation.

Dr. Ronald F. Scott, Research Assistant in Civil Engineering at M.I.T., and later Soils Engineer at the Arctic Construction and Frost Effects Laboratory, conducted the entire investigation described herein.

Mr. Rasmus S. Nordal, part-time Research Assistant in Civil Engineering, worked on the project during its second year and contributed to the construction and operation of the hydraulic analog computer.

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SUMMARY

This report presents the results of an investigation of the variables which affect the transfer of heat at the air-ground interface. Emphasis is placed on freeze and thaw penetration beneath airfield pavements, although the analyses presented are valid for any ground surface.

The investigation demonstrates that certain readily obtainable measurements may be utilized to predict the amounts of heat flow at the ground surface due to various atmospheric phenomena. Methods used to develop charts and diagrams for the convenient programming of a hydraulic analog computer are presented.

PART I - INTRODUCTION

1-01. Scope of Investigation. Calculations of depths of freezing and thawing below pavements are generally based on the variation of air temperature measured near the ground surface. In some instances, a semiempirical correction factor has been applied to correlate air temperature with ground surface temperature in order to predict subsurface temperatures. It was found at the start of this study that a thorough fundamental analysis of factors affecting heat transfer at the air-pavement interface had not been made. Thus it became desirable to initiate an investigation of these factors, the most important of which are radiation exchange and convection-conduction, and to evaluate their effects on the temperature regime in the ground.

This investigation demonstrates that certain readily obtainable measurements can be used to predict the amounts of heat flow due to various atmospheric phenomena. Effects of pertinent variables are analyzed. Methods used to develop charts and diagrams for the convenient programming of the hydraulic analog computer are presented. Although the analyses are valid for any ground surface, emphasis has been placed on heat transfer at airfield pavement surfaces since they are of principal interest in this study. In this investigation, all the possible atmospheric factors affecting heat flow at the ground surface have been reduced to two, i. e., heat transfer to the air by convection and conduction, and heat generated or lost by radiational processes. However, it will be found possible to use the air heat-transfer results for the determination of amounts of evaporation from a surface and resulting heat losses.

1-02. Scope of Report. Following this introduction, Part II details and discusses the contributing agents, emphasizing their relative roles in the heat economy of the ground surface. Those of greatest importance are taken up in the two succeeding parts. Part III considers the transfer of properties of the air by eddy processes. Since eddy diffusion in the air is not a simple, or well-understood phenomenon, this part is necessarily somewhat complex, although the results are presented as concisely as possible. Part IV summarizes previous work on radiation phenomena and presents results in a form suitable for computations. Since it is not generally possible to utilize meteorological data directly in the operation of the hydraulic analog computer, consideration has been given in Parts III and IV to averaging procedures, short cuts, and approximate methods of analysis. An explanation of the results of these studies is set forth in Part V.

PART II - FACTORS AFFECTING HEAT TRANSFER AT
THE EARTH'S SURFACE

2-01. General. In the following discussion, all the factors which cause heat to flow into or out of the ground surface will be analyzed. The flow is considered to take place into or out of a layer a fraction of an inch thick at the ground surface.

In effect, all the various factors affecting the heat transfer stimulate this surface layer to act as a heat generator, and the system of air and soil can be considered a passive accumulation of heat storage and transmission (by various means) upward into the air, and downward into the soil. The process is carried out by means of a forcing function of temperature at the interface. This is the physical model used by Lettau^{64, 65*} in his studies of temperature distribution and property variation in the soil and air.

Alternatively, the air-earth interface can be considered as a reference plane of infinitesimally small thickness through which various heat flows take place. Because of the plane's infinitesimally small thickness its heat capacity is zero, and as a consequence the sum of all heat flows into it must also be zero. This manner of looking at the problem gives rise to the 'heat balance' method of analyzing the heat movements involved and is the one considered in the subsequent work.

A list of the agencies which supply heat to or abstract it from this ground surface layer follows, with a short discussion of each item.

2-02. Solar Radiation. The sun's heat, consisting mostly of shortwave radiation, is the primary cause of all the other atmospheric phenomena. Because of the presence of the earth's atmosphere, there is always a scattering and reflection of the sun's rays in their passage through it, and this results in a diffuse shortwave radiation from either the sky or through clouds. Since the sun is an important source of energy on the earth, a great deal of solar radiation research has been carried out in recent years. Very little work has been done on investigation of solar radiation and cloud cover in arctic and subarctic regions, so that the curves and data presented here must be reviewed at a later date, when these researches have been carried out.

*Raised numerals refer to Bibliography at end of this report.

2-03. Longwave Radiation. Longwave radiation exchange occurs between the ground surface and the atmosphere. Longwave radiation arises from the fact that all bodies emit radiation according to their surface temperatures and properities. (Stefan's and Kirchhoff's Laws.) Since, according to Planck's radiation formula and Wien's displacement law, the intensity of radiation depends on the absolute temperature of the surface of the emitting body, it follows that the earth and atmosphere are comparatively longwave radiation emitters, while the sun, at its very high temperature, is primarily a shortwave emitter.

Shortwave radiation, while it is continually scattered and reflected in its passage through the atmosphere, is not especially subject to absorption. On the other hand, longwave radiation is continually absorbed, in several wavelength bands, and re-emitted by substances in the air. Because of the different wavelengths of the radiation being considered and the absorption bands due to the elements in the earth's atmosphere, the analysis of radiation is an extremely complex process, and must usually be investigated by leaning heavily on experimental data.

2-04. Soil Heat Conduction. Soil heat conduction pertains to heat flows by conduction through the soil itself, or through the pavement where it exists. This particular item in the heat budget is the one in which most interest is centered in this study. It is the attempt to estimate this flow and its effect on the soil temperature that causes the other terms to be studied. It is difficult to measure the heat properties of the soil, which vary considerably even in different zones of what is nominally the same soil, as well as in different soils, but if reasonable values are assumed for these properties it is considered that reliable results can be obtained.

The calculations which will be made do not take into account the heat flow due to ground water movement. Where the temperature of a mass of soil is changed by means of the flow of warm (or cold) water, a different term is given to the heat-transfer process, and it is called "advection," or "advective" heat transfer. Of course this takes place in nature. Temperature measurements made in the ground by Callendar and McLeod²³, and by Becker¹², indicate the effect of the percolation of water into the ground. It would be possible to take this factor into account, but it would be very difficult to calculate the amount of heat involved and the depth to which the water would percolate.

2-05. Heat Transfer to the Air. Heat flow from the surface to the air above takes place by means of conduction and convection processes. This is one of the most complicated transfer processes to deal with adequately, and many attempts have been found in literature to derive a sound theory for transfer processes in the air. Part III of this report presents the various aspects of this agency, but some mention of the problems involved will be made here.

In studies of heat transfer at the surfaces of bodies, the term "film transfer coefficient" is given to the conductivity of the air layer next to the surface of the body. It is well known that stationary air is a poor conductor of heat, and this thin, stagnant air layer offers a high resistance to the passage of heat through it, since heat transfer can take place through this layer only by molecular diffusion. Because the thickness of the stationary air layer at the surface varies with the conditions around the body, differences occur in the transfer coefficient. The problem studied herein considers horizontal surfaces only, in cases where heat can be transferred either to or from the air.

If the air is in movement past the surface of the body, and if a temperature difference between the surface and the air exists, the situation is altered. No longer does heat transfer take place by molecular diffusion and microscopic processes, but turbulence sets in, depending upon the conditions involved, and the exchange process becomes one of the movement of discrete masses of air carrying their properties with them and accelerating the heat-transfer process. Other factors affect this exchange too, particularly the surface roughness of the body.

Because of the nature of the earth's surface the turbulent type of air flow predominates, and its investigation is of fundamental interest in all micrometeorological studies. It becomes necessary to examine thoroughly the wind and temperature structure of the air immediately above the ground under all atmospheric conditions in order to ascertain the effects and magnitudes of the significant variables. The conditions which exist in nature at the earth's surface have been difficult or impossible to simulate in controlled laboratory tests, so that only limited data from wind tunnel or pipe flow measurements can be adapted to the needs of the meteorological problem. Most of the data utilized in succeeding analyses come from experiments conducted in the field, and it is only to be expected, as a natural consequence of the lack of field control of the significant variables, that the trends and rules and laws become difficult to establish. Because of this indeterminacy, the generally

high expense of outdoor establishments, and the demands placed upon the instruments working in adverse conditions, the lack of precise formulation in these analyses becomes explicable.

2-06. Evaporative Heat Flows. The natural processes of evaporation and condensation of water initiate heat flows at the surface of the earth. The quantity of heat involved must be examined by reason of its significance in the problem being studied, and because it is inextricably bound up with the analysis of other pertinent data. In this analysis it is necessary to arrive at theoretical or empirical methods for determining the heat movements due to all factors listed and discussed in this part of the report. The only way of separating and investigating certain factors is to consider situations where a sufficient number of significant variables have been measured at some location during a period of time, and to eliminate all the known heat flows. This implies that some heat flows are known, or can be measured. Accurate instruments have been devised to measure solar and net radiation at the earth's surface, but with the exception of some few locations where the flow of heat in the earth has been measured by heatmeters, all the other heat flows must be calculated from measured potential gradients and the appropriate properties of the medium concerned.

Since the soil at the surface of the earth in all but a few locations of economic interest is periodically moistened by rain, and since the air which passes over that surface will contain some water vapor, it follows that, depending on the relative temperatures of earth surface and ambient air, water will evaporate from, or condense on, the surface. The latent heat of evaporation of the amount of water involved will be abstracted from, or supplied to, the interface. Some correlation exists between this transfer process and those mentioned in Paragraph 2-05; this subject will be discussed in some detail in Part III.

As the study is from the overall viewpoint intended finally to deal with airfield pavements, which do not absorb water to the extent that the bare soil surface does, and which are presumed to be adequately drained, it follows that these studies are undertaken mainly as a corollary to the studies of convective heat transfer to the air. The heat loss from an airfield pavement owing to evaporation is likely to be a constant quantity depending on the thickness of water left forming a surface layer on the pavement each time it rains. The distribution of heat losses from this source during the year depends on the occurrence of the rainfalls. This aspect of evaporative losses is discussed more thoroughly in Part III.

2-07. Surface Water Freezing and Thawing. In a manner similar to that described in Paragraph 2-06, the melting or freezing of surface ice or water will involve a flow of heat. However, it is not proposed to consider this particular item at any great length because, where airfield pavements are concerned, the surface layer of ice or water will be so thin that no heat transfer of important magnitude is likely to occur.

2-08. Rain and Snow. The final factor in this analysis is the precipitation of rain or snow onto the surface of the ground or pavement at a temperature differing from it. It is assumed that in the case of an active airfield, the rain will be adequately drained off and that the snow will also be cleared from the surface in a relatively short time after deposition. Thus, although these items may assume a significant magnitude in short term studies of temperature fluctuations in the sub-soil over periods of a day or two, it is not likely that they will be important in investigations carried on over a whole year, or at least over the thawing or freezing period. These factors will have a larger effect on the heat flux by their alteration of the surface properties of the pavement or soil. Again, the snow will have but a short effect before its removal, but it may be possible that rainstorms rapidly succeeding each other will leave the ground surface moist for several days and have an important effect on its albedo and heat absorptivity. On the other hand rain does not usually fall on days of high irradiation, and when the irradiation does eventually occur, the moisture will rapidly disappear, having only the effect discussed under Paragraph 2-06. Thus only diffuse sky radiation will be likely to impinge on wet surfaces, and the loss of absorbed heat will be a small item in the total heat budget.

2-09. Data Limitations. As a matter of record, up to the time of the investigation reported herein no accurate heat-transfer data measurements have been made and recorded in arctic or even subarctic areas. All of the reliable information comes from tests made in areas of temperate to warm climate under convenient and comfortable weather conditions. This situation naturally increases the reliability of the data, while reducing the field of their applicability. Thus far, no knowledge has been accumulated on the effect of significant variables such as the drainage of soil properties due to freezing and thawing. It must, therefore, be taken as a temporary hypothesis throughout all of this work that calculations made on the basis of work carried out in temperate climates as in New Jersey, Nebraska, California, or in England, may be applied to airfield pavements in freezing and thawing seasons in

arctic climates as in Alaska. Not until adequate instrumentation is assembled and observations made in arctic areas to the same extent of detail as they have been made in these temperate zones will it be possible to test the empirical recommendations and charts presented herein.

2-10. Typical Calculations. In order to find the proportion of heat transfer supplied by each of the factors listed, and thus its relative importance, some typical calculations, made at various places on the earth's surface, will be examined. Certain authorities have prepared tables of the relative magnitudes of the different terms in the heat budget. Some of these data are shown in Table II-1 through II-4, together with information derived from literature cited. This information is used in subsequent parts of this report for the purposes of calculation. These data give an indication of the emphasis which may be placed on the various factors, and to what extent an error of estimate in any one will affect the total.

Table II-1
Daily Heat Fluxes, March 1948, in England (from Ref. 80)
Short Grass Surface
All values in cal/cm²/min

Latitude	Local Time	Solar Radiation		Longwave Outward Radiation	Heat Absorbed in Soil	Heat Used In Evaporation	Heat Removed by Turbulence
		Incoming	Reflected				
England	1030	0.715	- 0.123	- 0.191	- 0.057	- 0.120	- 0.224
52°	1230	0.770	- 0.196	- 0.196	- 0.105	- 0.126	- 0.215
	1530	0.336	- 0.153	- 0.153	- 0.017	- 0.097	+ 0.004
	1730	0.031	- 0.116	- 0.116	+ 0.048	- 0.029	+ 0.074
	1930	-----	-----	- 0.109	+ 0.063	- 0.023	+ 0.069

Tables II-1 and II-2 show variations of heat flux with time of day; Table II-3, with time of year; and Table II-4, the average annual condition. Note that in these tables the convention has been used that heat flux is positive when heat is flowing into the thin surface layer of the ground whether the flow is from above or below.

Attention is directed to the fact that the proportions of the heat budget vary considerably from place to place, even though the latitude is the same. Naturally the amount of evaporation which takes place at any given site, and, therefore, the

amount of heat used for the evaporation depends on water being available for the purpose. Evaporation will represent a greater proportion of the total heat flux at a site, for example, on the Washington or Oregon shores of the Pacific Ocean than it will in locations to the east of the Rocky Mountains at the same latitude. With this in mind, the tabulations may be examined.

Table II-2
Daily Heat Fluxes, 19 August 1953, in U.S. (calculated from Ref. 1).
Patchy Grass, Earth Surface. All values in $\text{cal/cm}^2/\text{min}$

<u>Latitude</u>	<u>Local Time</u>	<u>Net (Longwave and Shortwave) Outgoing Radiation</u>	<u>Heat Absorbed in Soil</u>	<u>Heat Used in Evaporation</u>	<u>Heat Removed by Turbulence</u>
Nebraska 42° 30'	0030	- 0.095	+ 0.070	+ 0.011	+ 0.014
	0230	- 0.094	+ 0.060	+ 0.011	+ 0.023
	0430	- 0.084	+ 0.070	+ 0.011	+ 0.003
	0630	- 0.044	+ 0.028	- 0.013	+ 0.029
	0830	+ 0.422	- 0.044	- 0.058	- 0.320
	1030	+ 0.676	- 0.114	- 0.193	- 0.370
	1230	+ 0.892	- 0.130	- 0.300	- 0.462
	1430	+ 0.759	- 0.118	- 0.359	- 0.282
	1630	+ 0.315	+ 0.016	- 0.162	- 0.169
	1830	- 0.012	+ 0.052	- 0.089	+ 0.048
	2030	- 0.087	+ 0.079	- 0.002	+ 0.019
	2230	- 0.092	+ 0.047	- 0.005	+ 0.050

Table II-3
Heat Fluxes, Spring, Summer, Fall and Winter, in Germany (from Ref. 2).
Surfaces, Short Grass. All values in $\text{cal/cm}^2/\text{day}$

<u>Latitude</u>	<u>Season</u>	<u>Time</u>	<u>Net Incoming Radiation</u>	<u>Heat Absorbed in Soil</u>	<u>Heat Used in Evaporation</u>	<u>Heat Removed by Turbulence</u>
Potsdam	Spring	Day	+ 64	- 6	- 60	+ 2
52°	Summer	Day	+174	- 20	- 104	- 50
	Fall	Day	+ 20	+ 6	- 29	+ 3
	Winter	Day	- 144	+ 43	+ 12	+ 89

Table II-4
Average Annual Heat Fluxes
Potsdam and Irkutsk (from Ref. 3)
All values in cal/cm²/ year

<u>Latitude</u>	<u>Place</u>	<u>Net Incoming Radiation</u>	<u>Heat Absorbed in Soil</u>	<u>Heat Used in Evaporation</u>	<u>Heat Removed by Turbulence</u>
52°	Potsdam	+ 19,819	- 181	- 20,025	+ 387
	Irkutsk	+ 19,928	- 560	- 11,561	- 7,807

In Tables II-1 and II-2 it will be noted that the outgoing radiation absorbs a large part of the total energy flux, amounting to 42% of the incoming radiation at noon, which cuts down immediately the heat available for the other processes. Generally speaking, the amounts of the heat budget removed by evaporation and turbulence to the air are equal in these locations, with the turbulence term predominating at midday. These two items account for 40 to 50% of the total outgoing heat flow. This leaves a flow of heat into the soil of only 10 to 20% of the irradiated heat flow. As it is this last small term with which the present discussion is ultimately concerned, it will be observed that a measurement of the amounts of the other terms must be made as accurately as possible for even moderately reliable estimates of heat flow into the ground. Small errors in estimation of turbulent and evaporative heat flows may result in large errors in the soil heat budget. However, as these terms for most of this work must be calculated over a whole year, or at the least over one freezing and thawing season, it is expected that many of the errors which will be unavoidably included will average out over the period. It is not possible, for short periods, to calculate the items closely enough for good results because of the very nature of the heat-transfer process in the atmosphere. Realizing that the values of soil heat conductivity and volumetric heat may be very approximate, extremely precise calculations of other items are unwarranted.

It can be seen that the total amount of heat involved in the budget at night is very small compared to that during the day, as the largest item, longwave outward radiation, is only 10 or 15% of solar (shortwave) radiation received at noon. The other factors are reduced correspondingly, and become even more difficult to take into account satisfactorily. The question of allotting the relative amounts of heat losses at night will be taken up in a later part, in which the weighting of day and night heat flows will be considered. These remarks are especially significant when the plot, Figure II-1, of the data given in Table II-2 is observed.

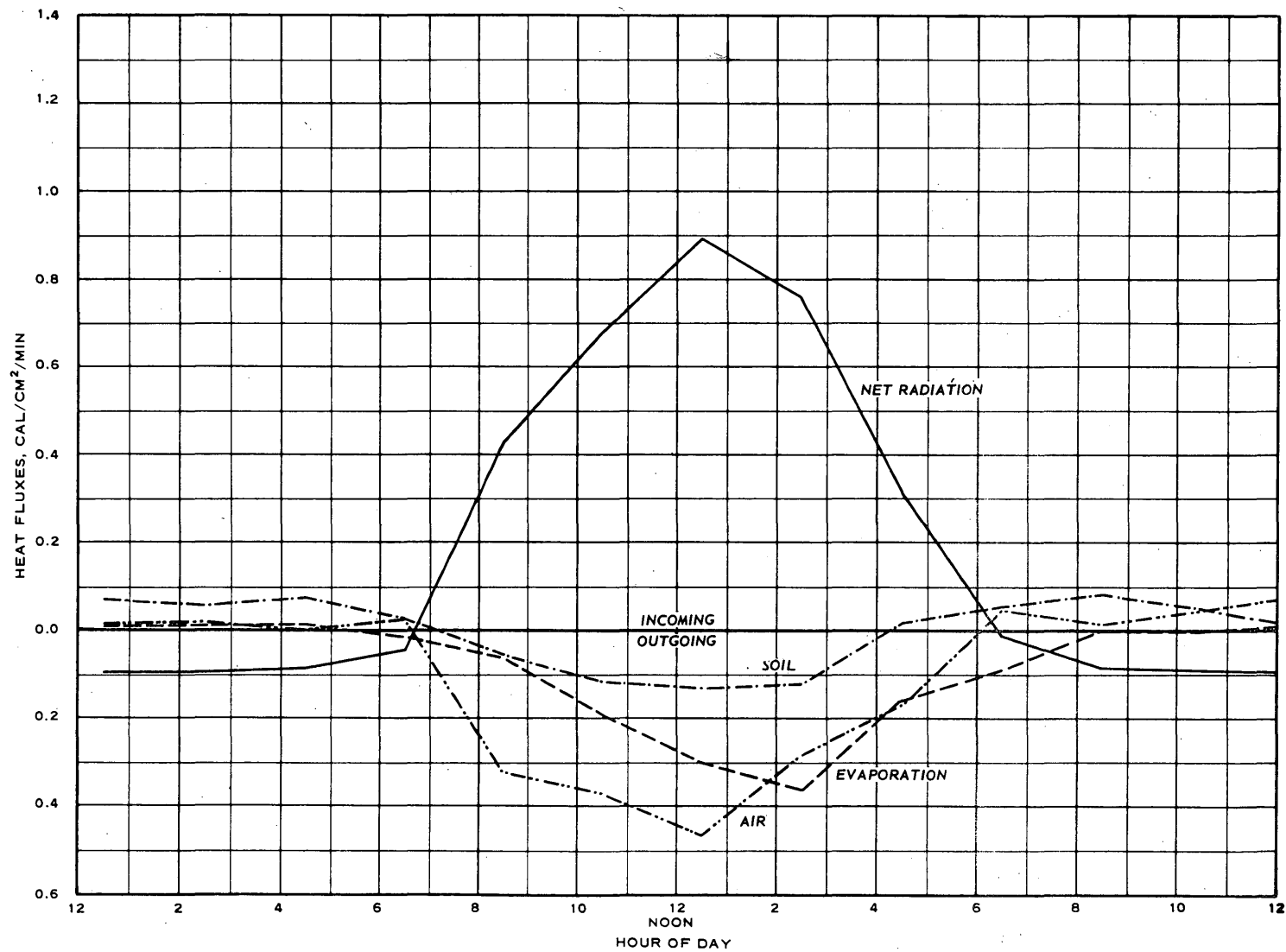


Figure II-1. Heat fluxes vs hour of day. 19 August 1953, O'Neill, Nebraska

In Table II-3, large seasonal variations in the heat-flow terms are noted, and their relative magnitudes fluctuate considerably. It is considered unusual that the data for Potsdam show such small amounts of heat loss to the air. The measurements and calculations were made by Albrecht in the late 1920's and it is possible that with an improvement of instrumentation and theory, different values might be found. Nevertheless, they may be taken as indicating the trends of heat flow. Thus during the summer at Potsdam, which has a damp, temperate climate, much of the sun's radiation energy is used in evaporating the soil moisture, while the ground, absorbing more of the radiation than at any other time of the year, acts as a sink, storing the heat, only to lose it again in the winter. It must be noted that the calculations have been carried out in this case for only one day in each of the seasons, and that they do not represent seasonal averages. This explains the fact that there is an apparent net loss of heat from the soil over the year in these figures. Probably, the ground is neither heating nor cooling on the average over the year.

Both the evaporation and turbulence factors show increases during the summer because of the higher temperature gradients involved, while condensation occurs during the winter when the air releases heat to the ground. It is thought that the actual quantity shown for the amount of heat transferred from the air to the ground for the winter day shown in Table II-3 is rather large and probably incorrect. In winter on this day the ground must have been colder than the air nearer the ground, which is a stable condition of the atmosphere, and it would, therefore, require a very warm wind moving at high velocities to agitate this layer and transfer the amount of heat indicated.

Table II-4 is presented to demonstrate the effect of location on the relative amounts of heat flow. It will be noticed that the amounts of incoming radiation are the same in both Potsdam and Irkutsk, but that in the humid location of Potsdam almost twice the amount of heat is lost by evaporation as in more arid Irkutsk situated in Central Asia on the shores of Lake Baikal. Similarly, in Irkutsk much more heat is removed by turbulence, possibly due to stronger winds, although it is felt that the negative heat flow due to turbulence in Potsdam is almost certainly far from correct. In the succeeding parts of this report the question of relative heat losses due to turbulence and evaporation is discussed more thoroughly. It is on the basis of that discussion that the doubts of the magnitudes of the Potsdam and Irkutsk data are expressed.

PART III - THE EDDY TRANSFER OF PROPERTIES OF THE AIR

3-01. General. This part of the report will be devoted to a study and consideration of the motion of the air in the air layers close to the earth's surface, that is, the lowest two meters of air, and the way in which this motion affects the heat economy of the surface.

To initiate the discussion it will be desirable to restate the problem, emphasizing those points which will be presented in more detail in the following pages. Soil and air meet at the earth's surface, and this surface plays an important part in heat transfer. The prime source of the heat system is the sun, and the dissipation of the energy of solar radiation in the first fraction of an inch of the soil surface creates heat in that thin layer. Some of this heat is conducted down into the earth and some is lost to the atmosphere in several ways, by longwave back radiation, by conduction and convection to the air, and by the evaporation of soil or surface moisture, as discussed in Part II.

Exclusive of radiation effects (which are discussed separately in Part IV), the way in which heat is transferred to the air depends on the movement of the air. If the air is quite still, then the heat must be transferred by conduction alone. This represents a lower limit of rate of transfer, as still air is a very poor conductor. However, if the air is not still, the thermal effects depend on the kind of motion.

A pure laminar flow will again result in pure conduction of heat, i.e., the transfer will take place essentially as if the air were still. From several causes, an agitation of the airflow called turbulence may take place which results in a mixing of the air by movements of larger masses. Thus a mass of air from one stratum with its concomitant properties is moved fairly rapidly into another stratum of differing properties, and the properties of the mass diffuse into the other layer to modify its characteristics in turn.

3-02. Definition of Properties. The properties referred to in this discussion will all be taken as 'conservative' properties of the medium considered. A 'conservative' property of a mass of fluid is one which can alter its concentration or distribution in the mass only by diffusion, whether molecular or turbulent, and not by processes such as chemical changes. Momentum, heat, vapor, and dust contents are

all considered to be conservative properties henceforward. A property such as the oxygen content of water would not be a conservative one, as it might combine chemically with a material in another different mass of water. The masses of air can be of all sizes from millimeter to mile dimensions in the atmosphere.

It will be obvious that this turbulent diffusion of properties will result in a more rapid transfer of them, and consequently in a more uniform medium, than will be obtained from molecular movements only.

3-03. Turbulent Flow. Turbulence in the airflow arises from several causes, and is still not very well understood although a great deal of study has been expended on it. If a long, sharp-edged plate is inserted in an airstream moving with uniform velocity and at a uniform temperature (the same as that of the plate), the resulting motion of the air next to the plate at different distances down from the leading edge can be observed and studied. Figure III-1 may be consulted here. If the plate is smooth, a laminar boundary layer (Prandtl, Blasius, etc., see Goldstein⁴⁵) is formed,

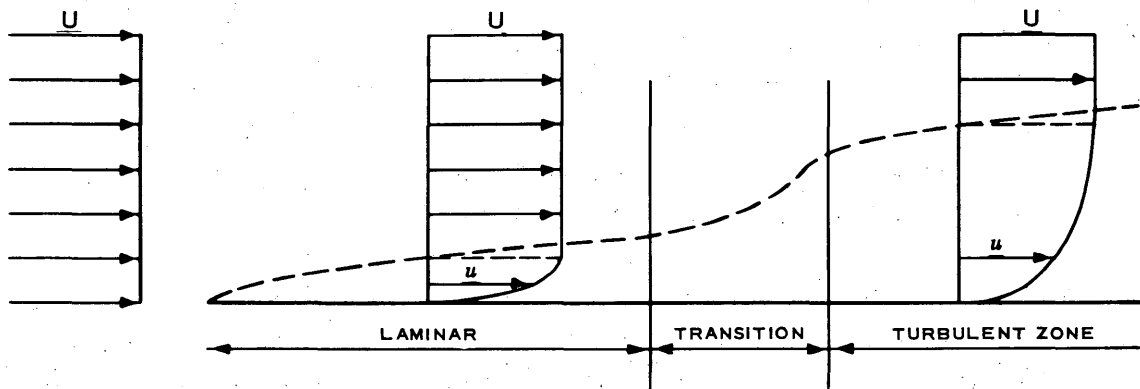


Figure III-1. Velocity profiles near a flat plate in uniform flow

in which there is a reduction in the air velocity as the surface of the plate is approached. This boundary layer, the depth of which is designated arbitrarily, grows in thickness with distance downstream until, at a particular point (depending on this distance, the wind velocity, and air properties⁹⁴) the smooth nature of the flow in the boundary layer ceases and the flow becomes turbulent. At this stage the velocity distribution in the now turbulent boundary layer changes, and it is found that the gradient of change in velocity with respect to distance perpendicular to the plate is everywhere less steep. The turbulent boundary layer increases in thickness with distance downstream from the leading edge and also with airflow velocity. This increase

has been investigated and expressed in the form of equations.⁹³ However, if the air-flow very close to the plate is examined, it will be found that a very thin layer of laminar flow still exists; the term 'laminar sublayer' is given to this small thickness in which momentum transfer still takes place by turbulent diffusion.

It has been said that the thickness attributed to the turbulent layer increases with distance, but obviously in such a case as flow over the earth's surface there is nowhere a situation where an airflow of uniform velocity comes in contact with a sharp edge equivalent to the laboratory experiment. Everywhere there will be boundary zones in which the air velocity depends on the height above ground surface, and these boundary layers can only be modified by contact with different 'leading edges.' Thus, for example, if the air moves uniformly over a sea for many hundreds of miles a certain profile of velocity distribution with height is likely to be established. The impingement of this wind on a shore in the form of gently sloping land, or even the extreme of a cliff, will produce an abrupt modification of the nature of the flow, which will persist in changing the former profile, until sufficient distance has been covered to establish a new profile. Because of the nonuniform nature of the earth and the many interruptions to the passage of the air, it is only to be expected that the wind profile will not remain of constant form for very great distances. Such a variability complicates all analysis procedures considerably. It is the principal reason for erratic results and the unsatisfactory nature of theory regarding wind structure over different parts of the earth.¹⁰³

From measurements made on the surfaces of different sizes of bodies (up to the dimensions of ships and airships) passing through fluid or gaseous media, it has been suggested²¹ that fully developed turbulent flow does not substantially exist at Reynold's numbers below 10^9 , measured from the starting point of any surface change of conditions. Such a value corresponds to distances on the earth's surface of 2-3 miles, and can therefore be achieved in only rare instances. Because of this, wind structure and temperature data taken at Seabrook Farms, New Jersey, U.S.A., which is referred to later, and in England by Pasquill⁸⁰, must be open to serious question, particularly in the latter case, as only a few hundred feet of clear unobstructed ground upwind existed. Such a criterion was evidently not considered by Pasquill, and there is little doubt but that his analyses refer to a developing boundary layer rather than a developed one.

So far, turbulence has been considered as beginning because of some inherent instability arising in the flow at a certain distance from the leading edge of a

smooth-surfaced plate. It has been found by Nikuradse⁷⁴ and others⁹⁴ that turbulence also develops because of the roughness of the surface over which flow is taking place. The onset of turbulence is governed in any particular instance by the air velocity and the roughness, in addition to the distance downstream.

In this case, a rough surface with a small size of protrusion causes an unevenness of flow next to it, probably only stimulating the turbulence to take place sooner than it would otherwise have done. If this type of flow with only small roughnesses is examined more closely, it is found that a laminar boundary sublayer still exists and effectively submerges the small projections.⁹³ In effect, the airflow, cushioned near the surface by this sublayer, does not know whether a rough surface or a smooth surface is present. The turbulence and the velocity profile which occur in this set of circumstances are independent of the actual size of the rough protuberances, providing they are smaller than the depth of the laminar sublayer. Thus equations have been established¹¹² which describe the velocity profile of flow over these surfaces, and, being independent of the roughness, contain no term to describe it.

If separate cases are imagined in which the dimension of the roughness increases,* the roughness will gradually begin to make its presence felt through the thickness of the laminar sublayer until in a final case it produces a substantial effect on the flow, which will depend on the magnitude of the projection, that is, on some dimension of the roughness. It is obvious now that any expression involving a velocity distribution and describing this type of flow must include a roughness term to effect the necessary modification.

Moreover, at any point on a rough plate, if the wind velocity is gradually increased from a very low magnitude, a gradual transition will take place in the nature of the flow above the point from that existing above a smooth surface to that existing above a rough surface. As the height of the laminar sublayer decreases it becomes, first, of magnitude comparable to the projections, and then of smaller magnitude. It would not be expected that any abrupt break or change in the nature of the flow would take place in such a slowly developing velocity pattern. Thus this flow transition will take place either by a variation in velocity, or by a variation in roughness. Equations have been developed to describe this rough surface type of flow.¹¹²

*It has been indicated previously that the various occurrences also depend on the wind velocity, and this applies because at any given distance from the leading edge of the plate, the boundary layer or sublayer will be thinner for a high velocity of air than for a low one. In consequence, the roughnesses are apparently enhanced.

It should be recognized that in the flow discussed above, there are two categories: transient flow, in which the velocity distribution is developing continuously and is a function of distance from a leading edge, or entrance; and steady state flow, in which no further change in the velocity profile takes place with distance in the direction of flow. The two types can be illustrated by reference to flow in long pipes and over very long level areas, such as the earth's surface. In both these cases there will be a distance at the beginning of flow over which a change in the velocity pattern takes place, say up to a length equal to 50 diameters in a pipe. For the rest of the distance, steady flow will hold with no change. The steady condition will have a predominating influence on flow calculations if the length over which it takes place is large compared to the developing range.

For comparison with data obtained for windflow over the earth's surface only those steady state equations which are applicable to flat plates will be used.

Thus far, turbulence has been dealt with as a function only of plate roughness, wind velocity, and length; that is, as a function of the physical dimensions of the surface over which the flow has been taking place. The name 'mechanical turbulence' may be given to this type of motion. But air motion can be developed without resorting to a bodily motion of the air masses concerned. If a plate which is not in an airflow is heated, the air in the immediate vicinity of the plate and above the plate is likewise heated, initially through molecular diffusion of heat in the layers next to the plate. The air nearest the plate, being heated more than air at greater distances, expands and becomes lighter than the air above it, which it therefore displaces by rising. The cooler, heavier air moves in next to the plate and in its turn is also heated and ascends. Thus a motion of the air arises due to its differential heating by the plate. Since the motion is in general irregular, it can be termed turbulence, or from the processes of convection which initiate it, 'convective turbulence'.

It will be recognized that the upward rising parcels of air carry their properties with them until they reach a stratum where, by gradual expansion and cooling, they mix with the surrounding air masses and distribute these properties to it. If the discussion is restricted once more to the horizontal plates, which are of the greatest interest herein, the motion involved in convective turbulence can be examined further. When a small plate is heated from below, it is obvious that the air above the center of the plate will increase in temperature more rapidly than the air along the edges, which is subjected to a loss of heat to the surrounding air body. Then

it is likely that the air above the center will achieve buoyancy first and break away from the plate, pulling the edge air in towards the center to fill the space, as in Figure III-2a. Eventually, if it is assumed that no bubbles of air break away, a steady situation will arise over a plate at a given temperature, in which air will flow over the edges of the plate, being heated as it goes, until it meets in the center, where the confluence rises vertically carrying its heat and causing the cyclic flow to continue. This motion will proceed indefinitely if an infinite source of air at a lower temperature than the plate is supplied to it.

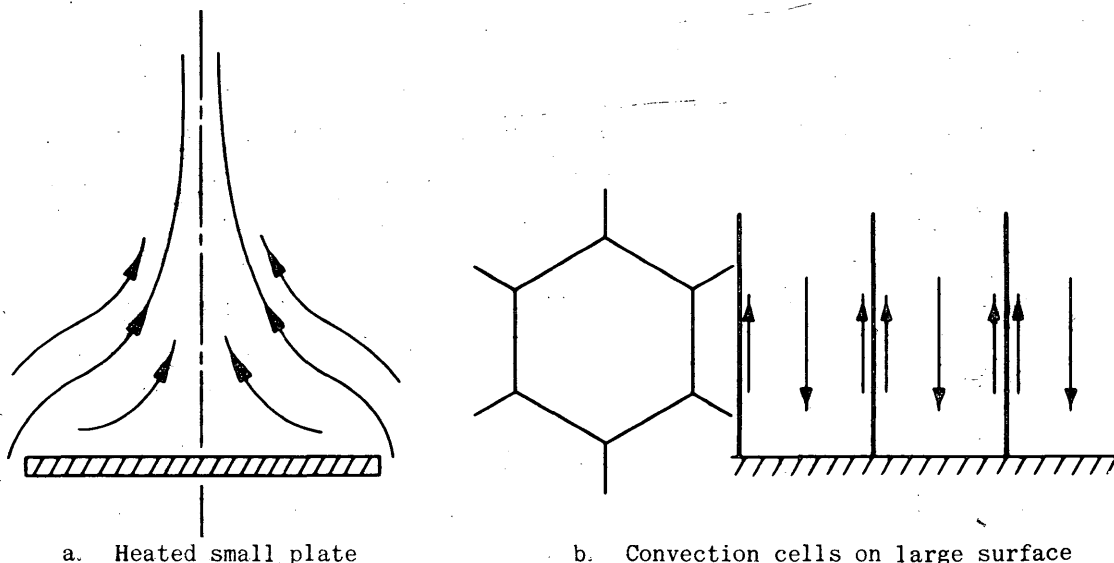


Figure III-2. Heat transfer between air and plane surfaces

This phenomenon has been well described in heat-transfer literature, and the relevant coefficients of heat transfer have been measured for various plate sizes.³⁶ The same problem, complicated by an airflow over the surface of the plate (a process known as forced convection), has also been extensively examined. Results will be found in the references mentioned above which concern themselves with plates of finite and generally fairly small dimensions.

When an infinitely large surface is considered, in other words, one which has a horizontal dimension large compared to the depth of the fluid above it, a more complicated situation arises, which was first investigated by Benard about 1900.¹³ In the case of a very large area warmer than the surrounding air, there can be no particular region which can be designated the center as in the simple, small, flat plate. Nevertheless, the air immediately above the plate must get warmer and therefore lighter than that higher up, which it must eventually tend to displace. What happens in such circumstances is that convection 'cells' occur, depending on the geometry of the

bounding layers, and the surface of the heated plane area becomes covered with a pattern of these cells, hexagonal in horizontal cross section, as in Figure III-2b. In each cell a circular motion takes place. The fluid being heated rises in one part of the cell due to its buoyancy and its place is filled with a cooler flow from another part of the cell.

The mathematical treatment of the process was undertaken by Lord Rayleigh⁸⁷ and further work has been more recently carried out by Southwell and his associates. Curiously enough these convection cells appear to exhibit a centrally rising warm column when the fluid is liquid with the colder liquid flowing down the sides of the cell, and a circumferentially rising warm current with a descending cool central column when it is air.¹²⁰ This may be because the viscosity of water and liquids decreases with temperature, while that of air and gases increases.

As a matter of interest, it may be noted that the 'soil polygons' in the arctic tundra may also be caused by slow convection motion of the soil during the thawing season.¹¹³

The whole phenomenon of Benard or convection cells is a very delicately balanced one, and occurs only under particularly specialized conditions. However, it has been surmised that certain types of cloud formations,¹⁰³ and some markings on the sea¹²⁰ may be due to such a convective motion. Thus, in all probability, under rare conditions, this form of motion takes place in the atmosphere. It is possible, then, that the motion described may yield results for heat-transfer processes near the surface of the earth (or ocean) under very calm conditions, and this may be an indication of numerical values at the lower end of the scale, that is, the vertical heat flow in the atmosphere when no horizontal wind is present.

By analogy with small plate heat flux data it follows that there will be a large increase in the rate of heat transfer at the earth's surface as soon as a wind begins to blow. The convective motion becomes forced convection, and convective turbulence is aided by mechanical turbulence. It may be assumed, with the exception of the possibility mentioned below, that pure laminar flow never takes place in the lower layers of the atmosphere. The flow will be either of the smooth or rough surface type; it remains to determine which of the two exists in a given situation.

Before a mathematical analysis and discussion of the convective heat-transfer problem is gone into, one further effect remains to be considered. Previously the

heat flow was considered to be taking place from the plate upwards, or in other words temperature decreased with height. However, with nighttime longwave radiational cooling of the earth's surface it is possible to achieve a situation in which the surface of the earth is colder than the air layers above, resulting in a temperature increase with height. In this case there is no tendency for the lower layers to become buoyant and break away from the surface; they are stable, in contrast to the instability which occurs when they are heated from below. Now, if there is no wind blowing, the heat-transfer coefficients in the lowest few centimeters of the air will tend to approach molecular values, and the heat flow, which is now due to conduction rather than convection, will be markedly inhibited, leading to the predominance of the longwave outward radiational heat loss in the heat budget.

If a wind exists along with the stable condition, there will be a strong tendency towards a damping of the turbulence, due to the inertia of the layer near the surface. It will be expected, therefore, that at some wind velocities a given roughness of terrain will give rise to rough surface flow during unstable temperature conditions, and smooth surface flow under stable conditions. This does not consider the possibility of laminar flow, which is only likely at extremely low wind velocities, in themselves very difficult to measure. In this way a logical picture of the apparatus of turbulence near the earth's surface can be drawn without resort to mathematical complexities. Without doubt, such a picture is oversimplified (for example, no definition of the degree of stability or instability has entered the discussion), and an artificial distinction has been made between mechanical and thermal turbulence, a distinction which cannot be made in nature but which serves as a qualitative basis for initiating a further quantitative study.

3-04. Study Procedure. The study will be carried out in the following way. Because of certain difficulties attendant on the derivation of expressions relating to convective transfer of heat in the atmosphere, which will be discussed in due course, it is proposed to examine the movement of water vapor or evaporation processes first of all. In so doing, the procedures are demonstrated which will in turn be applied to the other problem.

The evaporation of water from water and soil surfaces has commanded a great deal of attention in the last fifty years, and many empirical and semiempirical methods and equations have been proposed for dealing with the problem. These expressions and other methods utilized in the modern science subgroup of micrometeorology to describe the wind structure of the air near the earth's surface will be evaluated. The one

which is considered to be most correct and suitable among them will be chosen and applied to the analysis of data which have been obtained at one or two test areas in the United States. From the analysis, a quantitative chart is proposed for future use, and finally the modifications imposed by the consideration of convective heat transfer in the air will be introduced.

In order to test these theories, it will also be necessary to utilize data obtained on the heat budget of the ground surface. As it is not possible at the present time to make explicit measurements of the heat fluxes to the air and due to evaporation, it is necessary to adopt indirect means for their determination.

The indirect means usually consist of considering the heat economy of the ground surface plane, in which there can be no energy gain. Therefore, the sum of all heat flows into (or out of) this surface zone must be zero. By knowing or being able to calculate a sufficient number of the fluxes occurring, it is possible to deduce the unknown ones. This method is called the 'heat budget' or 'thermal energy balance' method, and was evidently first used by Schmidt in 1915.⁹⁷ It can be expressed for unit time in the form of an equation as

$$q_{ro} + q_{fo} + q_{co} + q_{eo} + q_{so} = 0 \quad (\text{III-1a})^*$$

in which all quantities are measured positively when heat is flowing into the surface, whether from below or above.

Generally speaking, it is possible to determine the quantity of heat flow taking place due to inward or outward radiation, by means of Gier Dunkle net exchange radiometers⁴⁴ and pyrheliometers. The heat flow in the soil can be obtained from heat-flow meters,¹¹⁶ or from measurements of the soil temperature profile and knowledge of the soil properties. In the latter case the temperatures and properties can be combined in layers where marked changes in the properties take place over the period of the observations. The total quantity of heat absorbed or given out is obtained for this period. The heat quantity in cal/cm²/sec will then be given by

$$q_{so} = \frac{1}{t} \sum_0^z (c \cdot \Delta T \cdot \Delta z) \quad (\text{III-2a})$$

*The meanings and dimensions of all symbols are given in Appendix A.

when z is the depth at which no temperature change takes place. This can be found by a method demonstrated by Hieronymus⁵⁰ if a long term calculation is being carried out. However, if the soil properties are uniform with depth or may be assumed uniform, then equation III-2a can assume the integral form

$$q_{so} = \frac{1}{t} \int_0^z c \cdot dT \cdot dz \quad (III-2b)$$

This is most readily evaluated by a planimeter on the depth-temperature-time curves. It will be noted that, if extremely accurate calculations are being made, the soil properties vary cyclically with time, as Lettau⁶⁶ has shown, probably due to fluctuations in daily moisture and temperature.

With the measurement of these terms, the heat budget expression becomes

$$q_{co} + q_{eo} = -(q_{ro} + q_{lo} + q_{so}) \quad (III-1b)$$

It was recognized by Bowen¹⁷ that, since q_{co} and q_{eo} occurred through the same processes of diffusion by turbulence, they would be likely to have a constant ratio to one another over a particular period, depending on the conditions then obtaining. Thus the term, 'Bowen ratio,' R , was given to the combination q_{co}/q_{eo} .

By substituting in equation III-1b, an expression

$$q_{eo} (1 + R) = -(q_{ro} + q_{lo} + q_{so}) \quad (III-1c)$$

is obtained. Bowen and Cummings and Richardson²⁷ investigated the dependency of R on the atmospheric conditions, and give equations for its determination. Later some discussion will be given on the values of R and the relation of q_{eo} to q_{co} in the analyses considered. It seemed more promising, however, and more in line with modern developments, to pursue the investigation from the point of view of turbulence and the eddy transfer of the properties of the air.

3-05: Evaporation Processes. Evaporation is the process by which a liquid changes to its vapor form. In so doing, the liquid requires an amount of energy, which is known as its latent heat of vaporization, in order to pass from a liquid to a gaseous state at the same temperature. If a liquid exists with a layer of gas above its surface containing vapor of the liquid, the amount of evaporation

depends on the direction and net rate of exchange of molecules of the liquid between the surface and the vapor. The number of molecules of liquid which leave the liquid surface per unit time depends on the temperature (or vapor pressure, which depends on temperature) of the liquid, and the number which return to it from the vapor depends on the temperature (or vapor pressure) of the gas. Thus the net rate depends on the vapor pressure difference between the liquid surface and the contiguous vapor. If the process is to continue, the molecules which leave the surface for the air must be removed from the air, or it becomes saturated. At saturation, the vapor pressures of the gas and the liquid become equal, and no more evaporation takes place.

It is this process of removal, or the outward diffusion of these molecules, which will be studied in succeeding pages. In general the diffusion will first of all be molecular, as the molecules pass through the thin laminar boundary layer, in which the vapor pressure gradient is linear, and then will take place more rapidly in the eddy processes of the turbulent layer. Because of the simple relationships existing in the laminar boundary layers, evaporation could very adequately be described by measuring the vapor pressure on each side of the layer, and finding the layer's thickness. However, the laminar layer is extremely thin, of variable thickness, and no techniques as yet exist for making vapor pressure measurements very close to a surface without affecting the surface conditions. The difficult task remains of adequately determining the equations which describe eddy conditions in the turbulent layer, and attempting to calculate the evaporation based on this knowledge. In the previous discussion of the transfer of masses of a fluid by turbulence due to the fluid motion, it was assumed that small displaced masses of the air will carry their properties with them, and that therefore the eddy diffusion of water vapor is equal to the eddy diffusion of momentum (or mass). Although some disagreement exists, this has been tested and proved by different authorities, and will be assumed to be valid in the following investigation. Consequently, an examination of the mechanism of mass transfer in the air will yield results of value to the calculation of vapor transfer. The properties of the air in motion will now be considered.

3-06. Momentum Exchange. The study of the velocity profile close to a surface began with Prandtl⁸³ in 1904 and has been extensively treated since that time. Many theoretical and empirical formulations, and combinations of the two, have been made to describe the flow under all conditions. Most work referred initially to adiabatic (strictly speaking, adiabatic conditions refer to temperature gradient of about -1° C./100 m which is negligibly small for most of the following work) airflow such as took place in carefully controlled laboratory tests, or in wind tunnels. Only fairly

recently has there been a development towards the introduction of terms to describe the thermal stratification of the air.

In the adiabatic equations two approaches can be recognized: the 'mixing length' concept which was introduced by Prandtl⁸⁵ and Schmidt,⁹⁸ and Sutton's¹⁰² 'continuous mixing' approach based initially on Taylor's¹⁰⁷ statistical theory of turbulence.

3-07. Mixing Length Approach. Briefly, the equations of molecular diffusion can be derived from Maxwell's considerations of the 'mean free path' of a molecule's motion. From this, expressions for viscosity, leading to the shearing stress across an interface (which represents mass transfer across the interface) can be developed in terms of the molecular paths. Prandtl extended this submicroscopic molecular idea of mean free paths to the macroscopic motion of air masses, moving in eddies, to derive an expression.

$$\tau = \rho l \sqrt{(w')^2} \frac{d\bar{u}}{dz} \quad (\text{III-3a})$$

The term $\rho l \sqrt{(w')^2}$ may be replaced by Schmidt's exchange (Austausch) coefficient or, as is more commonly used in recent analyses, $l \sqrt{(w')^2}$ is expressed by the kinematic eddy diffusion coefficient to obtain the equation

$$\tau = \rho K_M \cdot \frac{\partial \bar{u}}{\partial z} \quad (\text{III-3b})$$

where K_M is a vertical eddy coefficient of mass diffusivity. As all of the transfer process will be considered to take place in the vertical direction only, that is, a one-dimensional process, all values of K referred to will be vertical values. It will be noted that the transfer of properties across a plane of reference can be described by the product of a density, a diffusion coefficient, and a gradient of the property with respect to space. Thus the transfer of momentum is given by equation III-3b above, and the transfer of vapor and heat respectively will be given by equations

$$q_v = K_v \cdot \frac{\partial q}{\partial z} \quad ; \quad q_H = \rho C_p K_H \left(\frac{\partial T}{\partial z} + 1 \right) \quad (\text{III-3c})$$

which will be referred to again later, and in which q_v can be either a mass flow or a heat flow, if evaporation is the cause of the flow.

Various assumptions are required regarding the variation of the mixing length with height above the surface before practical expressions can be derived regarding the wind velocity profile with height. In the original theory the equations developed apply only to the adiabatic condition. No situations including temperature stratification can be treated by them.

The assumed distributions of the mixing length are as follows

$$l = k_0 z \text{ for a smooth surface} \quad (\text{III-4a})$$

$$l = k_0 (z + z_0) \text{ for a rough surface} \quad (\text{III-4b})$$

and the following equations result

$$\frac{\bar{u}}{u_*} = 5.5 + 5.75 \log_{10} \left(\frac{u_* z}{\nu} \right) \quad (\text{III-5})$$

from von Karman⁵⁸ for a smooth surface and

$$\frac{\bar{u}}{u_*} = \frac{1}{k_0} \log_e \frac{z + z_0}{z_0} \quad (\text{III-6a})$$

from Prandtl⁸⁴ for a rough surface.

These equations describe the profile of fluid flow velocity in ducts and tunnels, to which considerable experimental confirmation has been added.⁶² They represent a logarithmic relationship of velocity with height.

However, if the wind velocity above the earth's surface (from observations made in typical conditions prevailing over a twenty-four hour period⁸⁰ or ²⁹) is plotted versus height it will be observed that there are only one or two times when this logarithmic relationship holds. At other times, the velocity curves away from a straight line in a manner which seems to depend on the time of day. This variation is the effect of temperature stratification of the air, and must be accounted for in order to make the work applicable to heat flow in the atmosphere.

Before this is possible, it is necessary to consider how the physical condition of the air, including stratification, can be represented mathematically. Apart from Lord Rayleigh's investigations into the stability of Benard cells, this was first taken up by Richardson⁸⁸ in 1920. Previously, in this part of the report, mechanical

and thermal turbulences were mentioned: Richardson attempted to describe the relative magnitudes of dynamic and buoyant forces by means of the Richardson number

$$Ri = \frac{g}{T} \frac{\left(\frac{\partial \bar{T}}{\partial z} + \Gamma \right)}{\left(\frac{\partial \bar{u}}{\partial z} \right)^2} \quad (III-7)$$

Later papers^{29, 86} have discussed the value of Richardson's number. Priestley and Swinbank have noted in their analysis that a further term, to take care of what they define as "convective turbulence", should be added in the heat flux equation, but with respect to general usefulness, Ri has not been succeeded by a better expression. Perhaps its fundamental weakness is that the effects of buoyant and dynamic forces cannot be intrinsically distinguished, and the wind gradient with respect to height is affected by the thermal stratification. That is to say, if there is a large negative thermal gradient (temperature decrease with height), strong vertical mixing conditions will exist. This mixing tends to iron out variations in wind velocity with height giving rise to a small velocity gradient. Thus the larger (negatively) that $\partial \bar{T} / \partial z$ is, then the smaller $\partial \bar{u} / \partial z$ becomes, and the effect on Ri is emphasized by both terms. However, when $\partial \bar{T} / \partial z$ becomes positive (increase of temperature with height) there is a marked variation of velocity with height, and $\partial \bar{u} / \partial z$ also becomes large. Therefore it will be readily seen, before any numerical investigations are carried out, that Ri will tend towards high negative numerical values in unstable conditions, but will only have low positive values during stable conditions.

When $\partial \bar{T} / \partial z = -\Gamma$, the temperature gradient is equal to the adiabatic lapse rate, and neutral equilibrium conditions occur when $Ri=0$. In these circumstances it may be expected that equations III-5 and III-6 will hold in the atmosphere. This has been found to be the case. It has been seen that Richardson's number does not describe a range of atmospheric conditions with stable and unstable numerical zones equally divided by the neutral equilibrium value of zero. Consequently, it is difficult to use Richardson's number as a basis for discussing velocity equations, or as a parameter in expressions postulated to describe atmospheric wind and temperature fields.

The Richardson number has also been used in theoretical and experimental attempts to define the point of stability at which artificially induced turbulence is suppressed.^{89, 96} Values differ extremely, however, and only a very rough guide can be given to the development of any laminar layer in the atmosphere.

Gradually developing attempts were made by Sverdrup,¹⁰⁴ Holzman,⁵² and Rossby and Montgomery⁹² to incorporate stability expressions into wind velocity-height equations. E.R. Anderson, L.J. Anderson, and Marciano have analyzed these expressions in an excellent report.⁷ They came to the conclusion that an alteration of Rossby's formula by Holzman produces the best available wind law. All the laws considered can be re-expressed in order to give the amount of evaporation from a surface.

3-08. Continuous Mixing Theory. The alternate approach, the continuous mixing theory, was developed by Sutton¹⁰² as an extension of the statistical theory of turbulence originated by Taylor.¹⁰⁷ The mathematical difficulties of Sutton's work compelled him to use some expressions from Prandtl to derive equations for the eddy diffusion coefficient, and the final theory is not then limited to a statistical concept approach. The equations put forward to calculate evaporation derived from mixing length theory are all essentially 'point evaporation' expressions. That is, they recognize evaporation as a one-dimensional problem only, and consider that evaporation is the same at all points on a water or moist soil surface. They cannot be used to treat the diffusion of moisture from a finite area and do not describe the variation of evaporation and air vapor content properties with distance downwind from a leading edge of the evaporating surface.

The equations derived from the continuous mixing concept do recognize the variation in evaporation with downwind distance. Sutton's derivation led to the power law of wind distribution over a smooth surface:

$$\frac{\bar{u}}{\bar{u}_1} = \left(\frac{z}{z_1} \right)^{\frac{n}{2-n}} \quad (\text{III-8})$$

and it was observed,¹¹² by fitting the expression to experimentally observed atmospheric data, that n varied from a value tending toward 0.00 for extremely unstable conditions to a value tending toward 1.00 for extremely stable stratification ($n = 1.00$ would, of course, represent laminar flow). The equations derived by Sutton to describe evaporation were tested in a wind tunnel on small evaporating areas by various authorities¹¹² and were critically examined finally by Pasquill,⁷⁸ who modified them slightly to obtain good experimental verification.

Because of the experimental data which he was compelled to use in the derivation of his formulae, Sutton's expressions are applicable to smooth surface flow only. Extensions have to be made to the theory to include rough surface conditions. Tests

were made to see if the equations would apply in atmospheric conditions, but satisfactory results were not obtained. It becomes necessary to incorporate some roughness parameter into the velocity profile expression, and even to include some determination of whether smooth or rough surface flow was taking place. In 1949, Sutton¹⁰² introduced a concept of 'macroviscosity' which was intended to account for roughness effects in this formulae; this led to the development of new diffusion equations. In 1949, Deacon²⁹ proposed a general wind law based on the equation

$$\frac{\partial \bar{u}}{\partial z} = a z^{-\beta} \quad (\text{III-9a})$$

for rough surface flow and investigated the behavior of β at different stabilities. This equation and its development will be considered further in a later discussion.

Because of interest in evaporation from ocean surfaces, Sverdrup,¹⁰⁵ Montgomery,⁷¹ and Norris⁷⁵ investigated various forms of evaporation equations employing different velocity laws to integrate the equations through the different layers; their equations apply only to neutral conditions. Sverdrup makes a suggestion in order to find out whether the flow is taking place as smooth or rough surface flow: the appropriate equation is applied to the measured velocity at a particular height to calculate the shear velocity u_* at the surface. This is done for the velocity measured at several different heights. If the shear velocity calculated from each application remains constant, the correct equation is being applied; if not, a similar trial must be made with the other one. This of course can be carried out for adiabatic conditions only, as Sverdrup's conditions do not apply to thermally stratified systems.

3-09. Evaporation Formulae. Engineers and agricultural scientists have long been interested in evaporation losses from reservoirs and in evaporation and transpiration losses from soil and plants. Much of the earlier experimental work was done by them before the mechanics of turbulence was appreciated. Their results were generally given in the form of curves relating a wind factor to the wind velocity at a certain height. Hickox⁴⁹ summarizes the findings in an ASCE paper in which he presented data and formulae from experiments on the effect of many variables on pan evaporation.

The expressions will not be given here because of their inapplicability to evaporation from large surface areas.

In 1942, Thornthwaite and Holzman of the U.S. Department of Agriculture presented a development¹⁰⁹ of the mixing length theory, aimed at expressing evaporation as a function of wind velocity and humidity at two different heights in the atmosphere near the ground.

The equation they give is

$$E = - \frac{\rho k_o^2 z (\bar{u}_2 - \bar{u}_1)}{\log_e (z_2/z_1)} \cdot \frac{dq}{dz} \quad (\text{III-10})$$

and thus K_M or K_V , which they assume identical, is equal to

$$\frac{k_o^2 z (\bar{u}_2 - \bar{u}_1)}{\log_e (z_2/z_1)}$$

An integration of equation III-10 between two atmospheric levels yields an evaporation equation in which various constants are involved depending on the system of units. Because of the implicit use of the logarithmic wind law or the proportionality of mixing length to height in the derivation of the equations, they may be applied, strictly speaking, to times of neutral equilibrium only. However, Thornthwaite and Holzman use them under unstable conditions, and say they may be applied in the lower 16 ft of the atmosphere even under very unstable conditions. The logarithmic law of velocity breaks down a short distance above the laminar boundary layer; there is thus a limit to the heights at which it can be used.

Thornthwaite and Holzman point out that, as the law only begins at the lower level of the turbulent layer, this level may be taken as a "source surface" for turbulent transfer, and it is necessary to measure heights above this source surface rather than from the ground surface. The actual height of the source surface above the ground depends on vegetation, but if the ground is not under plant cover, it depends principally on the surface roughness. This is an "apparent roughness," as it will vary with time of day, and must be evaluated artificially from a consideration of the velocity distribution. The height of roughness is the level at which the wind velocity becomes zero by extrapolation and can be calculated by measuring the wind velocity at two different levels, provided the height of the source surface is known.

In the above reference,¹⁰⁹ a table is presented which shows the diurnal variation of the "apparent roughness" height (as calculated in the above-described manner), which is largest at night and smallest in the middle of the day. Thus the height is an indication of the stabilizing influence of temperature stratification at night when

strong radiational cooling builds up a cold surface layer to give rise to a large 'apparent surface roughness' from the measurements. Moreover, Thornthwaite and Holzman make the statement that 'the influence of stability on h_0 (which is the height of the apparent surface producing the observed velocity distribution) is not significant in determining evaporation,' even though no satisfactory evaluation of all factors can be made until such unplanned variables can be eliminated from the expressions.

The experiments of the U.S. Department of Agriculture Experimental Station at Arlington, Virginia, have led to studies made by the Johns Hopkins Laboratory of Climatology at Seabrook, New Jersey, since 1948, and a considerable amount of data has been amassed on the various atmospheric variables. These data are published at quarterly intervals and are a valuable source of information for analysis.

The final report¹¹² of the Lake Hefner studies, which presented the results of an investigation carried out on a small lake in Oklahoma, gives some valuable conclusions on the usefulness and applicability of the principal evaporation equations put forward and discussed in general terms above. It was ascertained that evaporation equations derived from Sverdrup's 1937 theory¹⁰⁵ and from Sutton's revised 1949 equations¹⁰² give the best indication of evaporation from the lake surface. Although doubts were raised regarding the validity of some of the parameters, it appeared in the report that Deacon's equation (III-9a) gave the best representation of wind distribution available.

3-10. Development of Deacon's Equation. Deacon's²⁹ equation can be used to develop eddy diffusion coefficients leading to point evaporation expressions and, on the basis of other preliminary work in the present investigation, was chosen as being the most convenient, while reasonably accurate, equation for use in the analyses following. Accordingly, a complete development of this equation will now be given to lead to the expressions which will be applied later. It is necessary to explain here the procedure which will be followed in order to obtain usable and reasonably correct data for the operation of the hydraulic computer.

In their quarterly bulletins, the Johns Hopkins Laboratory of Climatology supply information on hourly air temperature, wind velocities, and humidities at different heights above the ground surface for selected periods of about twenty-four hours duration, together with solar and net radiation data, soil temperatures and properties at various depths.

From the net radiation and soil temperature data it is possible to calculate the amount of heat which must be transported to or from the surface during each hour of the period involved by means of evaporation and by convection in the air. The equations which were referred to in the past few paragraphs and those which will be developed next will all be used for comparison purposes in calculating the amount of heat transferred due to evaporation or condensation. Attention can then be focussed on the object of most immediate interest, the convective heat transfer in the air.

Deacon²⁹ notes from observed variations of velocity gradient and potential temperature gradient $[(\partial T/\partial z) + \Gamma]$ with height, that Ri increased numerically almost linearly with height under both stable and unstable conditions. Therefore, in layers within a few centimeters of the ground surface, the influence of buoyancy forces on flow is slight (due to the initial low velocity of rising air masses), and the form of the velocity profile in this zone should not vary much with stability, but should approximate the logarithmic profile observed under neutral conditions. It was proposed by Deacon that for slightly greater heights the equation

$$\frac{d\bar{u}}{dz} = a z^{-\beta} \quad (\text{III-9b})$$

should be used, wherein $\bar{u} = 0$ at $z = z_0$

Integration gives

$$\bar{u} = \frac{a z_0^{1-\beta}}{1-\beta} \left[\left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right] \quad (\text{III-9c})$$

and on expansion

$$\bar{u} = a z_0^{1-\beta} \left[\log_e \left(\frac{z}{z_0} \right) + \frac{1-\beta}{2!} \log_e^2 \left(\frac{z}{z_0} \right) + \dots \right] \quad (\text{III-9d})$$

If this is postulated to hold in neutral conditions then it can be compared with Prandtl's surface equation

$$\frac{\bar{u}}{u_*} = \frac{1}{k_0} \log_e \left(\frac{z}{z_0} \right) \quad (\text{III-6b})$$

which is identical with equation III-6a for small roughnesses.

The two equations III-9d and III-6b are identical if

$$a = \frac{u_*}{k_0 z_0^{1-\beta}} \quad (\text{III-11})$$

and this may be substituted in equation III-9c to obtain the final velocity equation

$$\frac{\bar{u}}{u_*} = \frac{1}{k_0 (1-\beta)} \left[\left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right] \quad (\text{III-12})$$

for rough surface flow.

For the evaluation of this equation under a particular set of circumstances it is necessary to know z_0 and β . It should be noted in this context that the height z_0 is not a physical dimension of the system in the way that ϵ , Nikuradse's equivalent sand roughness, is, but that some experimental correlations between the two give an identity

$$z_0 = \frac{\epsilon}{30}$$

which can be used for comparing formulae, or visualizing a roughness height.

Actually it has been found, just as with Thornthwaite's and Holzman's expression, that z_0 varies with the stability conditions, but it is possible to force z_0 to remain constant by calculating its value under neutral conditions and using this value under all other stabilities. Neutral conditions are the only ones in which a known velocity law holds with reasonable certainty, and this law must be used to evaluate z_0 .

The procedure to be carried out for the determination of z_0 for a particular period is detailed in the following paragraphs.

The quotient of the wind velocities at two different chosen heights is obtained for each hour of the day and plotted versus the Richardson number for the same hour. From this graph an interpolation of the ratio of velocities obtaining at $Ri = 0$ can be made, that is, in neutral stability conditions. Under these conditions, the logarithmic law holds and can be applied.

Then

$$\bar{u} = \frac{u_*}{k_0} \log_e \frac{z}{z_0} \quad (\text{III-6b})$$

Consequently

$$\frac{u_1}{u_2} = \frac{\log_e \frac{z_1}{z_0}}{\log_e \frac{z_2}{z_0}} \quad (\text{III-13})$$

If two heights, for example 40 and 80 cm, are chosen, the right-hand side of equation III-13 can be evaluated for different values of z_0 and a plot of u_{80}/u_{40} versus z_0 can be made, which is applicable for all periods, provided that rough surface flow is taking place. This can only be ascertained by taking equation III-6b, rewriting it as

$$u_* = \frac{k_0 \bar{u}}{\log_e \frac{z}{z_0}}$$

after z_0 has been determined, and evaluating u_* for values of $\bar{u}$ obtained at several different heights at the time when neutral equilibrium exists. If the value of u_* remains consistent, rough surface flow is taking place; if it is not, a smooth surface formula such as III-5 must be applied. However, if other velocities occur during the day being analyzed, high enough that rough surface flow is almost certainly occurring, the velocities at neutral equilibrium must be ignored and the plot of u_1/u_2 versus Ri must not include them. The $Ri = 0$ value can still be obtained from the plot, however.

Values of u_{80}/u_{40} versus z_0 have been calculated. Results are shown below in tabular form and graphically in Figure III-3.

z_0	0.005	0.01	0.05	0.10	0.50	1.0	2.50	5.00
$\frac{u_{80}}{u_{40}}$	1.078	1.085	1.104	1.116	1.158	1.189	1.252	1.334

Now that the value of z_0 has been determined for the period under consideration, it remains to compute the value of β at every hour of the period, so that the velocity profile may be fitted by equation III-12. If, in the determination of z_0 as before, two heights are considered, the ratio of velocities at the two heights can be obtained from equation III-12 as

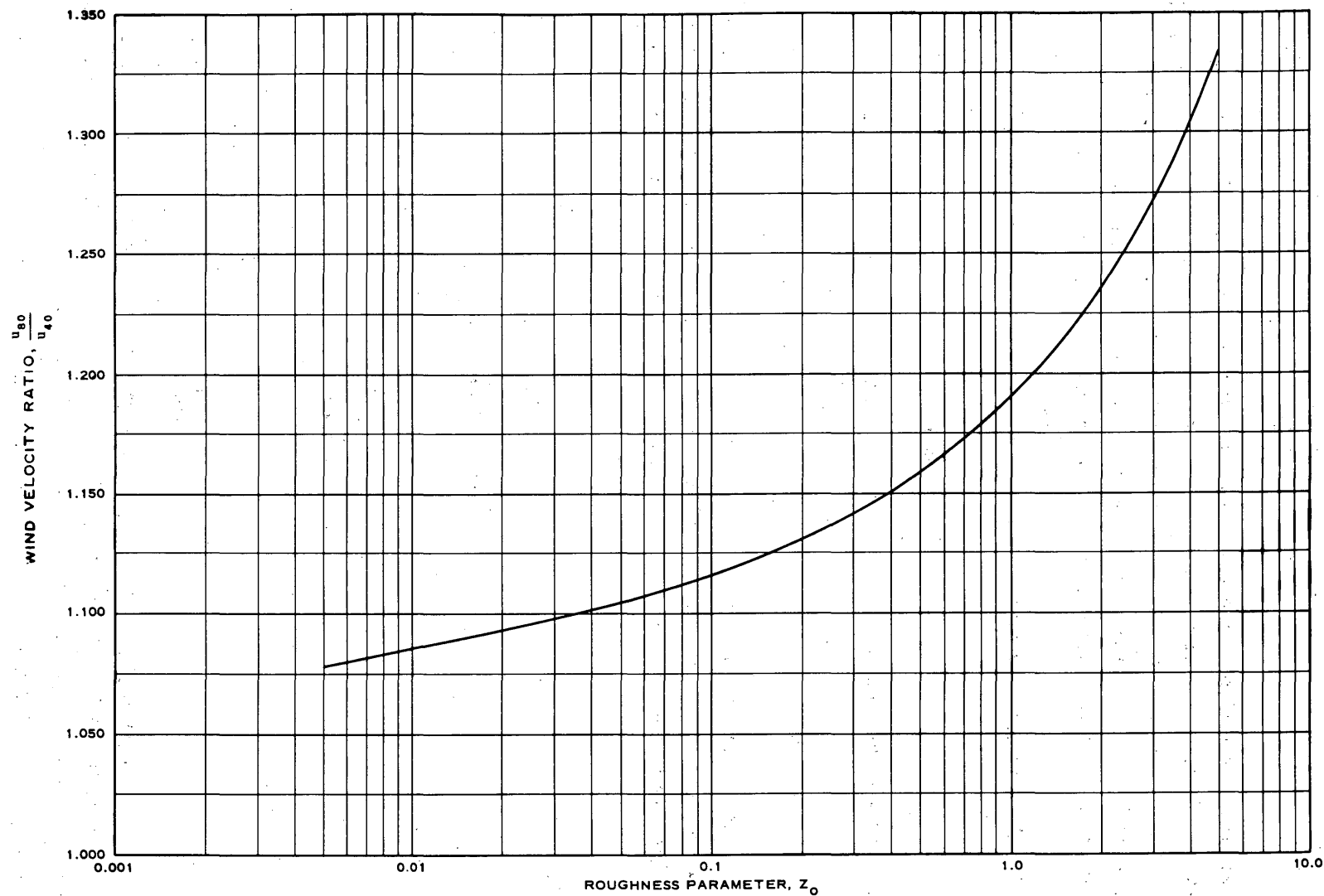


Figure III-3. Wind velocity vs roughness parameter (equation III-13)

$$\frac{u_1}{u_2} = \frac{(z_1/z_0)^{1-\beta} - 1}{(z_2/z_0)^{1-\beta} - 1} \quad (\text{III-14})$$

This expression can also be plotted graphically for different β values, as z_0 is now known. At each hour of the period, the curve can be entered with the known u_1/u_2 value, and β found. Thus it is possible to fit Deacon's equation to the velocity profile existing at any time. It remains to derive an expression from equation III-12 describing the vertical exchange of momentum or vapor.

For the vertical eddy diffusion of momentum, we have the equation

$$\tau = \rho K_M \frac{d\bar{u}}{dz} \quad (\text{III-3b})$$

and for the vertical eddy diffusion of airborne material, such as water vapor,

$$E = \rho K_V \frac{dq}{dz} \quad (\text{III-10b})$$

where q is the vapor concentration at z in appropriate units. As was pointed out earlier in references 22 and 101, there is evidence to show that $K_M = K_V = K$ is a reasonable assumption, so that it is sufficient to calculate K from Deacon's equation.

From equation III-3b

$$K = \frac{\tau/\rho}{d\bar{u}/dz} = \frac{u_*^2}{d\bar{u}/dz}$$

and from III-12

$$\frac{d\bar{u}}{dz} = \frac{u_*}{k_0 z_0} \left(\frac{z_0}{z} \right)^\beta$$

then

$$K = k_0 z_0 u_* \left(\frac{z}{z_0} \right)^\beta \quad (\text{III-15})$$

3-11. Analyses of Data for K Determination. It is thought advisable to state the problem again so that a clear view of the data which are required may be obtained. The hydraulic analog computer depends for its solutions on the heat-transfer conditions which are simulated at the air-ground interface. At any particular location

these conditions must be obtained from normal weather bureau information taken at a station as close as possible to the site. This information, which is supplied in U.S. Weather Bureau National Monthly Summaries,¹¹⁴ consists of daily air temperatures, humidities, and wind velocities measured at a particular height above the ground, together with measurements of sunshine duration or cloudiness (which will be dealt with in Part IV). Additional long-term information, which may be used generally for design purposes, may be found in publications such as the Climatological Atlas of Canada,¹⁰⁸ the Climatic Atlas of the United States,¹¹⁷ or further U.S. Weather Bureau publications such as reference 111.

From these data curves must be prepared showing the hourly, daily, weekly, or monthly solar radiation input, convective and evaporative heat-transfer rates, and longwave radiational cooling amounts, either explicitly or implicitly in terms of ground surface temperatures. With the exception of the solar radiation input, no information of this nature has yet been prepared in suitable form; that is the purpose of this section of the investigation. The most convenient way of presenting material is in the form of charts or nomographs relating pavement (or ground) surface transfer coefficients to wind velocities measured at conventional weather bureau stations. The heat-transfer coefficient can then be used in conjunction with the temperature at that height to give the correct heat flow when the computer supplies the surface temperature.

In order to analyse data to obtain the desired charts, however, it is necessary to consider the calculation of evaporation heat losses from the surface. The past discussion has mentioned the equations which have been put forward from time to time to describe the point evaporation process. It was considered that a comparison of these equations was in order to determine the most accurate or, if no marked discrepancies appeared, the most readily applicable. This comparison was made on the basis of data published by the Johns Hopkins Laboratory of Climatology and by the University of Wisconsin. The data were taken at O'Neill, Nebraska, on 18-19 August 1953, as part of the Great Plains Turbulence Field Program,¹ conducted by the U.S. Air Force Cambridge Research Center.²⁸

Because of the limitations imposed on the logarithmic law of wind velocity distribution, it cannot be strictly applied to atmospheric conditions other than those obtained at neutral stability, but Holzman⁵² has given as an evaporation equation a development of equation III-10 which, he states, can be used for the determination of evaporational heat losses under most conditions of the atmosphere other than very stable. The equation has been used in the following form:

o

$$E = \frac{\rho k_o^2 (q_1 - q_2) (u_2 - u_1)}{(\log_e z_2/z_1)^2}$$

As Pasquill⁷⁹ points out, this equation can be made more convenient for computing purposes without much loss in accuracy by writing

$$E = \frac{\rho k_o^2 \left(1 - \frac{u_1}{u_2}\right) u_2 (q_1 - q_2)}{(\log_e z_2/z_1)^2}$$

or

$$E = \frac{k_o^2}{RT} \frac{(1 - u_1/u_2)}{(\log_e z_2/z_1)^2} u_2 (e_1 - e_2)$$

If the value of u_1/u_2 is found for neutral equilibrium conditions and assumed to hold approximately throughout a 24-hour period, a simple expression

$$q_e = A u_2 (e_1 - e_2) \quad (\text{III-16a})$$

is obtained in which q_e is the evaporative heat loss in appropriate units, per unit time, and A is a numerical constant or assumed constant which is calculated for the period considered. Then the evaporational heat losses can be calculated quite easily from the wind velocity at one elevation and the vapor pressure at two.

Pasquill has examined the reliability of this formula in effectively adiabatic conditions by comparing its results with those found by measurement of the actual loss of moisture from the soil by weight, and has concluded that it has value under these conditions. The equation is included here in its present form for use in all conditions only because of its simplicity of application.

The second method to be used is one derived from the Deacon equation III-12, in which the eddy diffusivity coefficient is calculated, and then applied to the vapor pressure gradient over the range in which it applies. It is generally recognized that the flux of a property is the same across any plane above the surface of the generating plane to heights of a few meters; that is, τ the shearing stress (momentum flux) or q the quantity of water vapor (evaporative flux) does not vary with elevation.

The equation is therefore of the form

$$q_e = B K_V (e_1 - e_2) \quad (\text{III-17})$$

where

$$B = \frac{L}{(z_1 - z_2) RT} \quad (\text{III-18})$$

The K_V value is calculated for the height difference z_1 to z_2 or averaged over it, and is computed for every hour of the observation. The atmospheric stability is therefore taken into consideration.

The University of Wisconsin data¹ published on the Great Plains Turbulence Project separates the heat flows due to evaporation and convection by means of a Bowen ratio, but the report does not say how the ratio was computed. In the course of this investigation an inquiry was made, which yielded the following reply:

'In regard to your inquiry as to how we measured the Bowen ratio - it is defined as follows:

$$B = \frac{L}{E} = \frac{C_p \rho}{l_T 0.621} \frac{\partial T / \partial z}{\partial e / \partial z}$$

0.621 = ratio of molecular weight of water vapor to that of dry air

C_p = specific heat at constant pressure - .241 $\frac{\text{cal}}{\text{gm}}$ deg for mixing ratios observed at O'Neill

l_T = latent heat of vaporization at temperature, T . (Approximately 588 cal/gm at wet bulb temperatures observed at O'Neill.)

ρ = pressure at O'Neill; usually between 940 and 950 millibars

e = vapor pressure in mb

E = heat used for evaporation

L = convective heat transport

The psychrometer equation may be written $e = e_w - K (T - T_w)$.

e_w = vapor pressure at wet bulb temperature

T_w = wet bulb temperature

T = dry bulb temperature

K = psychrometric constant = $\frac{C_p \rho}{0.621 l_T}$

Taking differences we may rewrite the psychrometric equation as

$$\Delta e = \Delta e_w - K \Delta (T - T_w) \quad (1)$$

Noting that $\Delta e_w = k \Delta T_w$ where k is the average slope of the vapor pressure-temperature curve in region ΔT_w , we may rewrite equation (1) as

$$\Delta e = k \Delta T_w - K \Delta (T - T_w) \quad (2)$$

Since $T - T_w$ is by definition the wet bulb depression, we may write equation (2) in the following form:

$$\Delta e = k \Delta T_w - K \Delta \text{dep}$$

Using the identity $\Delta T_w = \Delta T - \Delta (T - T_w) = \Delta T - \Delta \text{dep}$, we may obtain Δe and hence the Bowen ratio from measurements of the absolute wet bulb temperature and gradients of temperature and wet bulb depression.

Whenever the Bowen ratio approaches minus one, no solution for L and E can be obtained from the energy balance equation. All that can be said is that E and L are equal in magnitude but opposite in sign. We have found that whenever the Bowen ratio is between -0.6 and -1.4, our computed values of E and L would be very improbable if not impossible."

The heat flow due to evaporation given in the above-cited report has also been considered in the comparisons made in this study as indicated in the tables and charts included in this report.

These comparisons were all carried out before knowledge of the Lake Hefner reports⁷ came to hand, and only subsequently were they considered.

The final Lake Hefner report analyses the effectiveness of several formulae in predicting evaporation from a small lake in Oklahoma, based on the actual evaporation from the lake as computed from measurements of inflow, outflow, precipitation and other pertinent data. In this report, two formulae were found to be the most reliable, one due to Sutton¹⁰² and one from Sverdrup.¹⁰⁵ Sutton's formula, derived principally from continuous mixing methods, takes into account the area of the surface and the

diminution in evaporation with distance downwind. It is extremely complicated to evaluate and is otherwise not suitable for estimating one-dimensional evaporation from a large flat area. The equation due to Sverdrup was derived from a consideration of a two-layer model including both a laminar and a turbulent zone and is therefore more realistic in its assumptions than most of the other work which has been done on the basis of a turbulent layer existing down to the surface. However, Sverdrup's expression, which was derived to apply to water surfaces, utilized the vapor pressure of the water surface to calculate the evaporation. The vapor pressure method required knowledge of the surface temperature but in most investigational periods during the Great Plains Turbulence Project, and the Johns Hopkins Seabrook Project, no surface temperatures were measured. It is therefore difficult to utilize Sverdrup's formula in its evaporation form. It is not known, in any case, from what level in the soil the water is evaporating.

If the equation is considered, on the other hand, from the point of view of the evaporation being equal to a conductivity term times a gradient, then it will be possible to separate the two. The overall coefficient of diffusivity which will be part molecular and part turbulent in its nature can then be calculated. The equation given in the Lake Hefner report refers to the vapor pressure at the zero and eight meter levels, but it can be written down in a more generalized form as

$$E = \frac{\rho k_o u_* (q_o - q_z)}{\log_e \left(\frac{z + z_o}{\delta + z_o} \right) + \frac{k_o \delta u_*}{D}} \quad (\text{III-19})$$

for rough surface flow.

This gives

$$E = \frac{k_o u_*}{RT} \cdot \frac{(e_o - e_z)}{\log_e \left(\frac{z + z_o}{\delta + z_o} \right) + \frac{k_o \delta u_*}{D}}$$

then

$$q_e = C K_V (e_o - e_z) \quad (\text{III-20})$$

where

$$C = \frac{L}{zRT} \quad (\text{III-21})$$

and

$$g_v = \frac{k_0 u_* z}{\log_e \left(\frac{z + z_0}{\delta + z_0} \right) + \frac{k_0 \delta u_*}{D}} \quad (\text{III-22})$$

to enable comparison to be made with equation III-17).

In these equations δ is the thickness of the laminar sublayer. It has been defined by von Karman⁵⁸ from measurements as

$$\delta = \frac{30\nu}{u_*} \quad (\text{III-23})$$

The friction velocity u_* can be obtained for these calculations either from the measurements of shearing stress at the ground surface made by Vehrencamp¹ with the University of California group at the Great Plains Project, or from the application of equation III-12 which, as shown later, gives good correlation with Vehrencamp's studies. It will be necessary to obtain z_0 , the roughness parameter, by the same method as before, by using the logarithmic law in neutral equilibrium conditions.

In spite of the apparently good correlation obtained in the lake Hefner report between actual evaporation and that computed by equation III-19, a study of Sverdrup's original paper shows it to be based on some very questionable assumptions, as for example, the use of a logarithmic law for the humidity profile, and the postulation that it holds right down to the upper surface of the laminar boundary layer, in spite of Wust's¹²¹ investigations and conclusions. Also the whole formula is based on the logarithmic law of wind profile, which does not hold under all stability conditions.

However, this equation has been shown by the Lake Hefner studies to yield useful results. It should therefore be considered in all studies where the ground surface temperature and/or vapor pressure is measured. It may be noted here that an application of Sverdrup's methods to wind and temperature information might give information on the thickness of the various boundary layers under different conditions, which could be carried out in further studies.

3-12. Calculations of Equation Reliability. It is now proposed to carry out an analysis of some of the Great Plains Turbulence Project data on the basis of the equations III-16a and III-17 and the results of the University of Wisconsin group using the Bowen ratio only. No information on vapor pressures or temperatures at the ground surface is available to enable equation III-20 to be tested. The period chosen was

from 1900 hours on 18 August 1953 to 2000 hours on 19 August 1953. This particular day was chosen because the data on solar radiation, net radiation, heat flux in the ground, temperature and velocity profiles were apparently taken by the University of Wisconsin¹ and Johns Hopkins University (hereafter referred to as UW and JH) independently at the site; since it is necessary to use information provided by the Johns Hopkins University report in order to evaluate the effect of the equations, it was desirable that the overlapping data provided by the two institutions should conform as closely as possible. Of all the days on which complete test runs were made, the two reports coincide most closely on 18-19 August 1953 to give the best prospect of a satisfactory analysis.

Information regarding instruments and methods used at the sites can be obtained from the publications mentioned. It is desirable to give a short description of the methods by which some of the pertinent data were obtained. The UW obtained the soil heat conduction values by means of strip resistance thermometers connected in series, so that the average (with respect to space) temperature at a given depth was measured, while JH measured soil temperatures at point depths by means of thermocouples mounted in copper tubes. Both methods necessitated some disturbance of the soil. The heat capacity of the dry soil for the site was found by electric calorimetry, and then the capacity at any depth could be calculated from the moisture content of selected soil samples taken at appropriate times. Later in the series of tests, a heat meter was installed by the UW group and was found to give very promising results. In general, heat meters disturb the temperature regime of the ground initially by being placed on or in it and over a longer period through their interference with moisture migration. Thus they may be expected to give information adequate for one or two day's examination of soil heat properties, but over a longer period a one-dimensional heat-flow condition through the meter can no longer be looked for.

The UW data on temperature and vapor pressures were observed at 44 and 88 cm above ground level by mounting the temperature and humidity sensing devices on a lift which automatically reversed the positions of the instruments every ten minutes in order to cancel out errors due to differences in the sensitivity of the instruments. Hourly averages were constructed on the basis of sixty readings an hour.

On the other hand, JH obtained temperature and humidity information by means of fixed level devices at 10, 20, 40, 80, 160, 320, and 640 cm above the ground surface. Twenty air temperatures were recorded in the five minutes centered at 35 minutes past each hour and were averaged to give the reported values. In the JH publication only wind velocities are given which are used for all information involving wind speed.

In certain cases, such as wind velocity gradient and absolute value of temperature, which are required for a computation of the Richardson number, the values were taken from the JH data at heights of 40 and 80 cm, and were used to give a value for the number over the intervening distance. This number may then be used to describe or extend the information obtained for UW data at heights of 44 and 88 cm. It is not considered that the deviation of this number from the one which would hold for the 44-88 interval has any significant effect on the evaluation. It is known and has been widely reported that the Richardson number varies with height, an undesirable feature which has been recently investigated by Businger,²¹ but it is not thought that the small variation which would occur by interpolating the JH data to find the Richardson number appropriate to the 44- to 88-cm level was worth while.

Some remarks are perhaps in order with regard to the columns of calculation in Table III-1. The first two columns are self-explanatory, and the net radiation is

Table III-1
Analysis of Data from O'Neill, Nebraska, 18-19 August 1953

No. of Observations	1 Time	2 Net $q_r + q_l$	3 q_s	4 Heat Available for q_e, q_c	5 NT_{44-88}	6 $\frac{NT}{Az} \times 10^{-3}$	7 T	8 u_{40-80}	9 $\left(\frac{u}{Az}\right)^2$	10 $Ri \times 10^{-3}$	11 $\frac{u_{80}}{u_{40}}$	12 $\rho c_p \times 10^{-3}$
1	19-20	-0.087	+0.065	+0.022	0.349	7.83	291	28	0.49	53.8	1.204	0.291
2	20-21	-0.085	0.068	0.017	0.476	10.72	289	28	0.49	74.2	1.360	0.293
3	21-22	-0.088	0.067	0.021	0.397	8.92	288	26	0.423	71.8	1.197	0.294
4	22-23	-0.092	0.047	0.045	0.362	8.13	289.5	37	0.857	32.2	1.213	0.293
5	23-24	-0.100	0.084	0.016	0.294	6.58	289.5	42	1.125	19.9	1.188	0.293
6	24-01	-0.095	0.070	0.025	0.284	6.36	287	28	0.490	44.3	1.195	0.295
7	01-02	-0.085	0.066	0.019	0.283	6.33	285.5	24	0.360	60.3	1.250	0.297
8	02-03	-0.094	0.060	0.034	0.206	4.58	285.5	33	0.681	23.1	1.215	0.297
9	03-04	-0.087	0.058	0.029	0.190	4.22	285	31	0.601	24.2	1.210	0.298
10	04-05	-0.084	0.070	0.014	0.171	3.79	286	38	0.903	14.42	1.212	0.296
11	05-06	-0.085	0.048	0.037	0.176	3.90	287	51	1.625	8.20	1.208	0.295
12	06-07	-0.044	0.028	0.016	0.136	2.99	287.5	49	1.500	6.79	1.205	0.295
13	07-08	+0.202	-0.003	-0.199	-0.143	-3.35	291	58	2.10	-5.38	1.196	0.291
14	08-09	0.422	-0.048	-0.378	-0.313	-7.22	294	61	2.33	-10.36	1.188	0.288
15	09-10	0.629	-0.285	-0.344	-0.441	-10.12	296.5	77	3.71	-9.02	1.216	0.286
16	10-11	0.677	-0.114	-0.563	-0.421	-9.66	297.5	75	3.52	-9.04	1.193	0.285
17	11-12	0.916	-0.141	-0.775	-0.604	-13.83	298.5	78	3.81	-11.94	1.194	0.284
18	12-13	0.892	-0.130	-0.762	-0.596	-13.63	299.5	69	2.97	-15.03	1.183	0.283
19	13-14	0.730	-0.099	-0.631	-0.460	-10.56	299	70	3.06	-11.32	1.192	0.283
20	14-15	0.760	-0.118	-0.642	-0.531	-12.16	300	66	2.72	-14.60	1.178	0.282
21	15-16	0.547	-0.057	-0.490	-0.376	-8.65	300.5	66	2.72	-10.40	1.187	0.282
22	16-17	0.315	+0.016	-0.331	-0.237	-5.48	299.5	63	2.48	-7.23	1.187	0.283
23	17-18	0.103	0.192	-0.295	0.005	+0.01	298.5	56	1.96	+0.02	1.180	0.284
24	18-19	-0.012	0.052	+0.040	0.226	+5.03	296	29	0.53	+31.5	1.180	0.286
25	19-20	-0.087	0.049	0.038	0.561	+12.63	290.5	28	0.49	+87.0	1.286	0.292

taken from the reported data of UW. They correspond closely to the UW results for the times involved, as shown in the plot Figure III-4. It was thought necessary to check the figures recorded in column 3 from the UW data, by means of plotting the soil temperatures (from JH) with depth, measuring the area by planimeter, and calculating the heat absorbed in the soil per hour from equation III-2a. Some typical profiles are

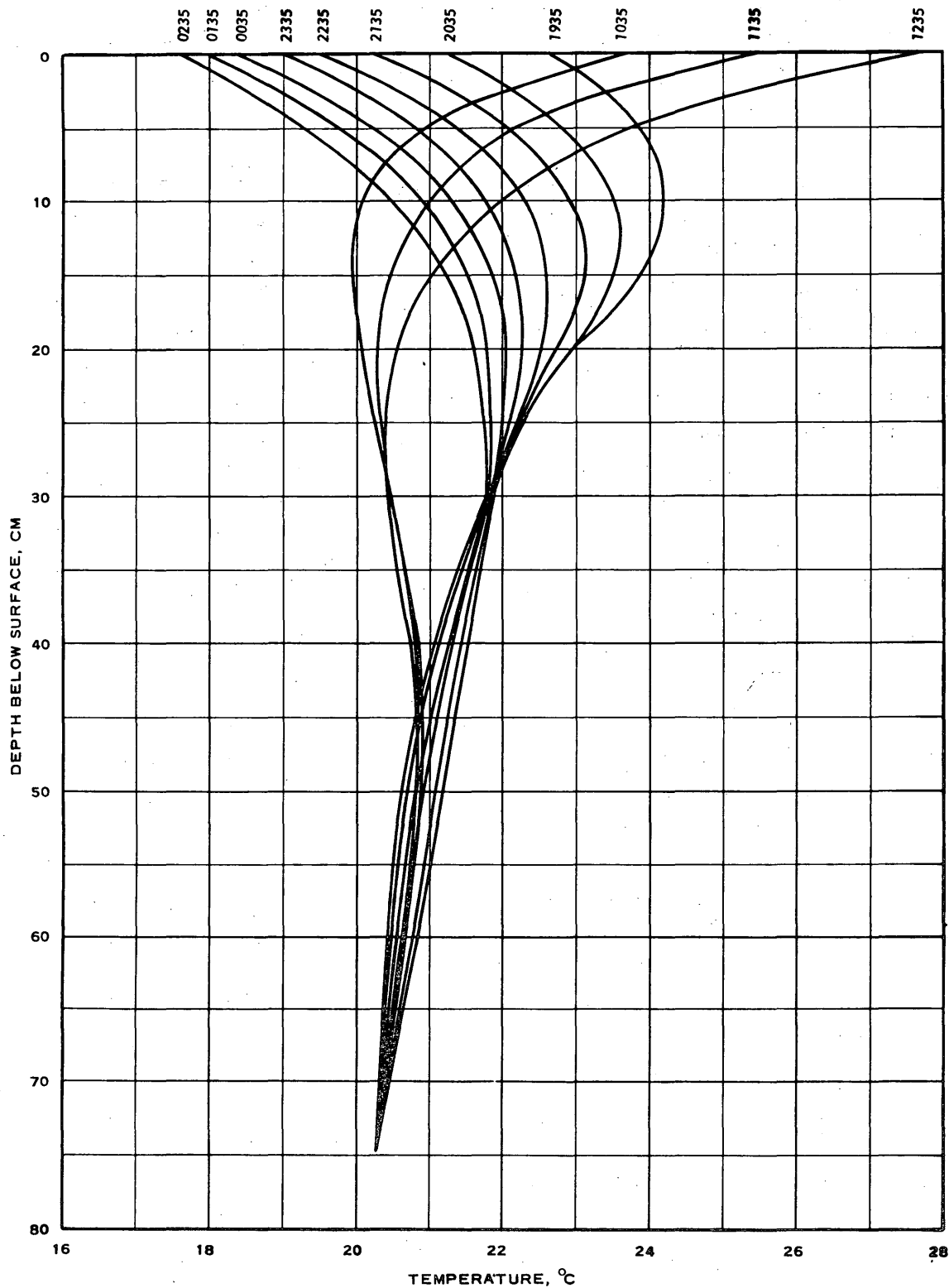


Figure III-4. Soil temperature vs depth at selected times
18-19 August 1953, O'Neill, Nebraska

shown in Figure III-4 which was used only for those times at which a reasonable extrapolation to a surface temperature was possible. Thus the hours in the middle of the day and early afternoon must be excepted. From the comparison of UW and JH information it was considered that the UW data was more reliable.

From columns 2 and 3, it was possible to compute the amount of heat which must have been removed by evaporation and turbulent convection to the air. This is shown in column 4, which will be used in order to partition the heat flows based on the various formulae. Column 5 merely presents the temperature difference between the 44- and 88-cm levels from the UW report, and this is reduced to the potential gradient $[(\partial \bar{T}/\partial z) + \Gamma]$ in the sixth column. In columns 7, 8, and 9 data are taken from the JH report to give the absolute temperature, the difference in wind velocities at the 40- and 80-cm levels, and the square of the wind velocity gradient with respect to height. These figures are then all used to compute column 10, which is the Richardson number according to equation III-7.

For both the Holzman's equation III-16a and Deacon's equation III-17 it is necessary to find the value of u_{80}/u_{40} holding at adiabatic conditions. Because of the possibility of introducing errors by taking the value at which a Richardson number of zero occurs, it is desirable to make a plot of u_{80}/u_{40} versus Ri . Consequently values of u_{80}/u_{40} are shown tabulated in column 11, and a plot, Figure III-5, has been drawn. From the plot, u_{80}/u_{40} was chosen equal to 1.200 at equilibrium conditions. This value has been entered in Figure III-3 to give a height of roughness of $z_0 = 1.25$ cm for use in the Deacon formula.

In order to ensure that rough surface flow is taking place, it is necessary to apply equation III-6b to different velocities at different elevations at times when adiabatic flow is taking place. This has been done in Table III-2 at 0630 hours and it will be seen that there is not doubt of the consistency of the value of the friction velocity u_* obtained. The times when conditions are adiabatic will be seen to be 0630, 0730 and 1730 hours on this date (19 August 1953) and the velocities at all levels at 0630 hours are lower than those holding at the other times. Consequently, if rough surface flow is taking place at 0630 hours, as has been shown to be the case, then rough surface flow must exist at still higher wind velocities. The calculations can be carried on after this demonstration.

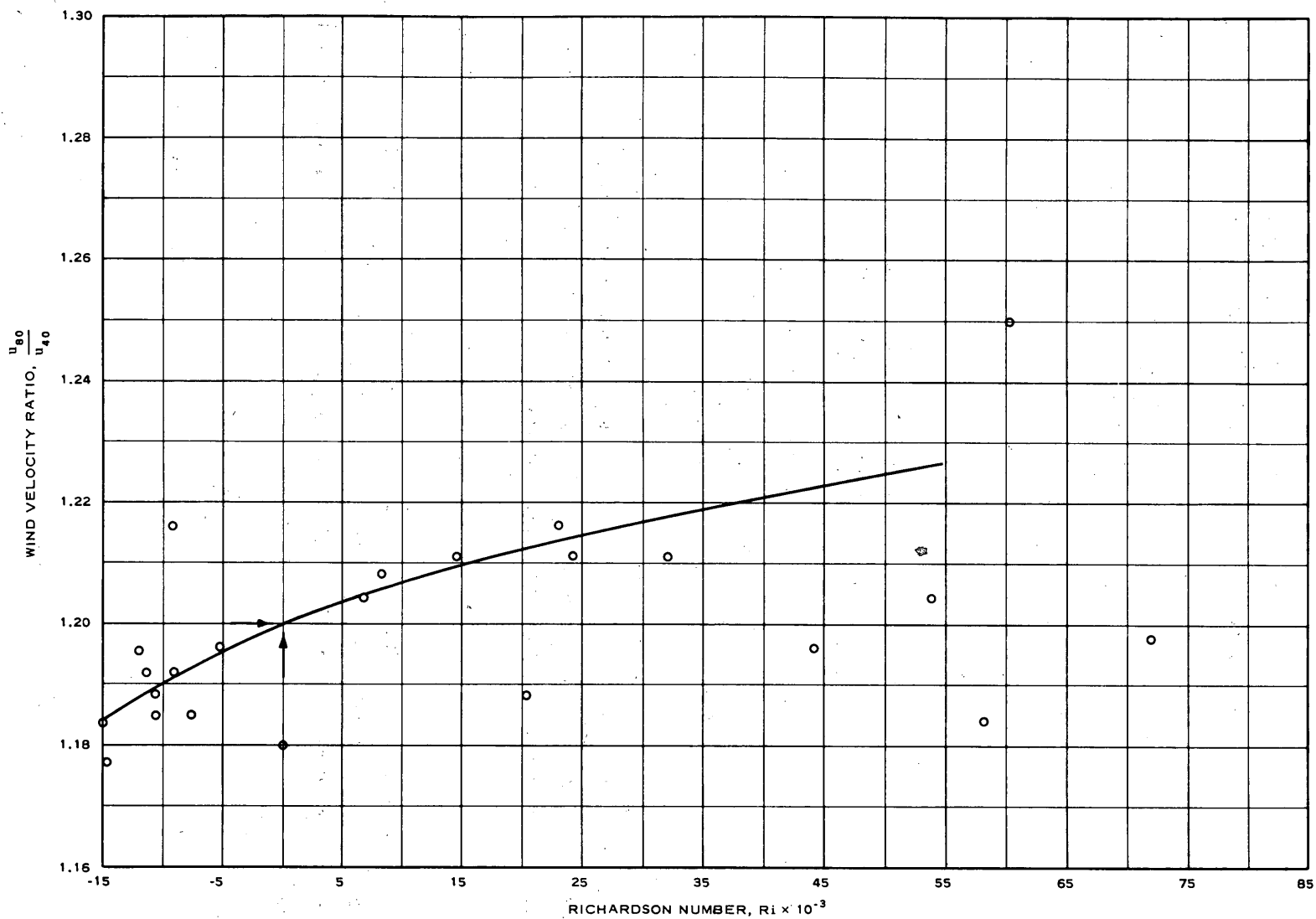


Figure III-5. Values of wind velocity ratio vs the Richardson number. 18-19 August 1953, O'Neill, Nebraska

Table III-2

Determination of Smooth or Rough Surface Flow at
Equilibrium Conditions at O'Neill, Nebraska, 19 August 1953

Equation III-6b is used to determine u_*

Values taken at hour 06-07

$z_0 = 1.25 \text{ cm}$

z	$\frac{z}{z_0}$	$\log_e \frac{z}{z_0}$	u	u_*
40	32	3.466	241	27.8
80	64	4.159	290	27.9
160	128	4.852	332	27.4
320	256	5.545	381	27.4
640	512	6.238	446	28.6

In Holzman's formula, equation III-16a, the coefficient A can now be computed.

$$A = \frac{k_o^2 (1 - u_{40}/u_{80})}{RT (\log_e z_{80}/z_{40})^2}$$

Where $u_{40}/u_{80} = 1/1.20$ at equilibrium conditions = 0.833. The usual values are taken for the other constants. T is the average absolute temperature prevailing throughout the day (the variation in the absolute temperature is relatively small) in the air. It is found that equation III-16a becomes

$$q_e = 1.346 \times 10^{-3} u_{80} (e_{40} - e_{80}) \quad (\text{III-16b})$$

Then in Table III-3 it remains to note the values of u_{80} and $(e_{40} - e_{80})$ in columns 2 and 3 respectively. A multiplication involving equation III-16b gives rise to column 4 which represents the evaporation heat loss calculated from Holzman's formula III-16a.

Table III-3

Heat Loss due to Evaporation, from Equation III-17

No. of Observations	1 Time	2 u_{80}	3 $e_{40} - e_{80}$	4 q_e	5 β	6 $\left(\frac{z}{z_0}\right)^{1-\beta}$	7 u_*	8 $\left(\frac{z}{z_0}\right)^\beta$	9 K_{M60}	10 q_e
1	19-20	166	-0.064	+0.0143	0.99	1.039	15.46	46.2	357	+0.0125
2	20-21	106	0.071	-0.0101	0.70	3.194	5.08	15.0	38	-0.0015
3	21-22	158	0.014	-0.0030	1.01	0.962	15.37	49.9	384	-0.0030
4	22-23	213	0.020	-0.0057	0.97	1.123	19.16	42.9	412	-0.0045
5	23-24	265	-0.028	+0.0100	1.03	0.891	27.1	54.1	732	+0.0113
6	24-01	172	-0.043	0.0100	1.02	0.926	17.33	52.0	450	0.0108
7	01-02	120	-0.094	0.0152	0.89	1.530	9.13	31.4	143	0.0075
8	02-03	187	-0.054	0.0136	0.97	1.123	16.9	42.9	363	0.0109
9	03-04	179	-0.052	0.0125	0.98	1.080	16.5	44.4	367	0.0107
10	04-05	219	-0.046	0.0136	0.97	1.123	19.83	42.9	425	0.0109
11	05-06	296	0.020	-0.0080	0.99	1.039	27.85	46.2	642	-0.0071
12	06-07	290	0.036	-0.0140	0.99	1.039	27.25	46.2	629	-0.0126
13	07-08	356	0.148	-0.0708	1.01	0.962	34.95	49.9	873	-0.0707
14	08-09	387	0.099	-0.0515	1.03	0.891	39.65	54.1	1073	-0.0575
15	09-10	435	0.253	-0.1483	0.97	1.123	39.1	42.9	838	-0.1139
16	10-11	468	0.295	-0.1857	1.02	0.926	47.1	52.0	1225	-0.1933
17	11-12	483	0.468	-0.3040	1.02	0.926	48.7	52.0	1267	-0.3165
18	12-13	450	0.398	-0.241	1.05	0.823	48.7	58.3	1421	-0.3000
19	13-14	438	0.398	-0.235	1.02	0.926	43.9	52.0	1144	-0.2420
20	14-15	438	0.472	-0.278	1.06	0.792	47.3	60.7	1436	-0.3595
21	15-16	422	0.441	-0.250	1.03	0.891	43.4	54.1	1176	-0.2745
22	16-17	401	0.273	-0.1472	1.03	0.891	41.2	54.1	1115	-0.1617
23	17-18	370	0.220	-0.1096	1.05	0.823	40.0	58.3	1167	-0.1370
24	18-19	191	0.135	-0.0347						
25	19-20	126	0.057	-0.0097	0.82	2.008	8.07	23.9	96	-0.0030

In order to evaluate the evaporation from Deacon's equation III-17 it is necessary to compute the average value of the eddy diffusion coefficient K_V over the interval 40 to 80 cm. It was considered sufficiently accurate to find K_V prevailing at 60 cm and to apply this to the 40- to 80-cm vapor pressure difference. K_V is given by equation III-15 which involves u_* , and u_* can be calculated from equation III-12, given the velocity at a particular height. It is seen that the procedure brings in the quantities

$$\beta, \left(\frac{z}{z_0}\right)^{1-\beta}, \text{ and } \left(\frac{z}{z_0}\right)^\beta$$

and these are tabulated in columns 5, 6 and 8 which give rise to u_* in column 7, leading to the evaluation of K_V in column 9.

The stability coefficient, β , is found most readily from a graph of the equation III-14 which is shown in Figure III-6. The terms $(z/z_0)^{1-\beta}$ and $(z/z_0)^\beta$ may be charted versus β , or may be obtained by using a 'log-log' slide rule.

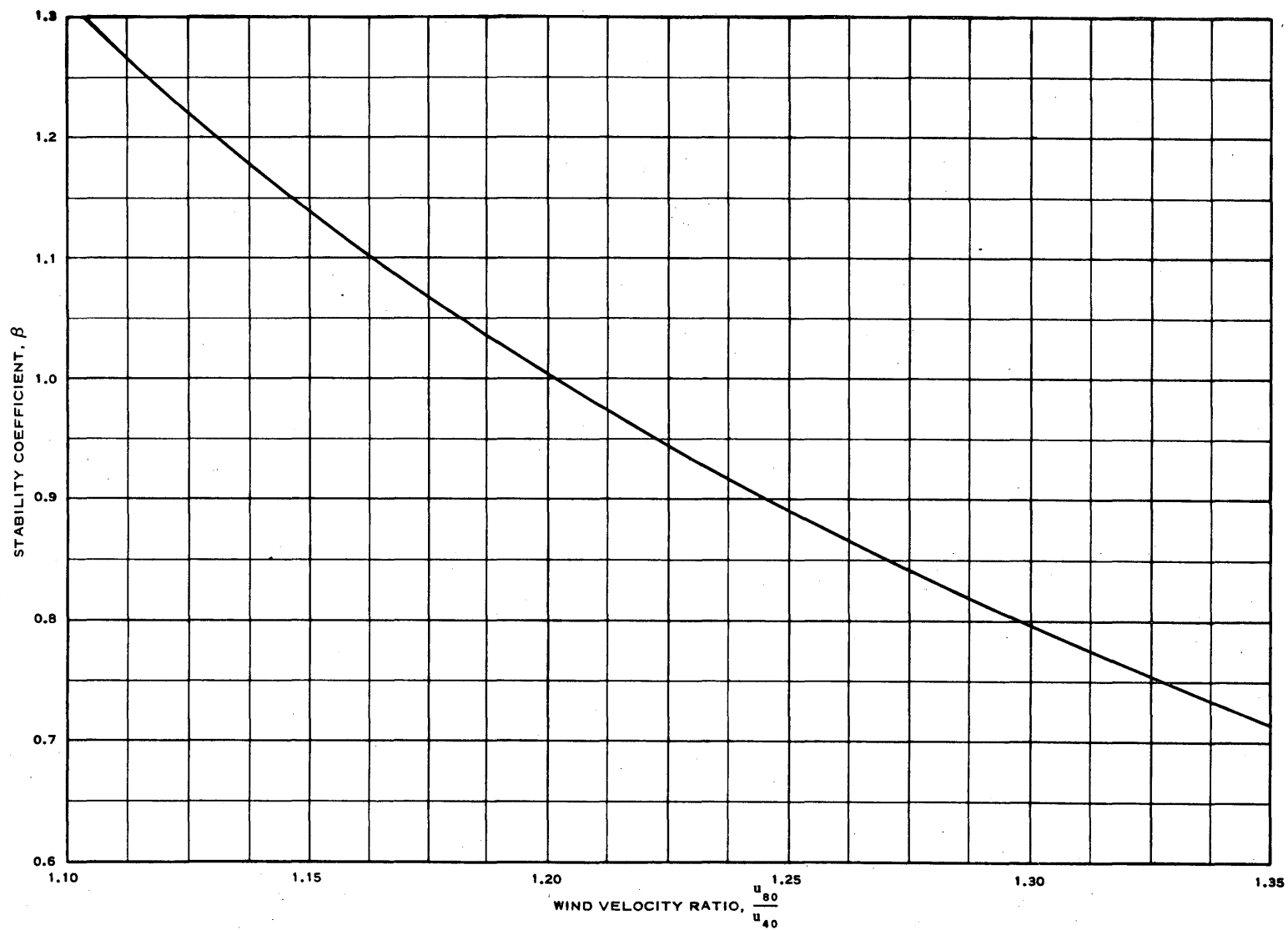


Figure III-6. Values of stability coefficient vs wind velocity ratio for $z_0 = 1.25$ cm. 18-19 August 1953, O'Neill, Nebraska

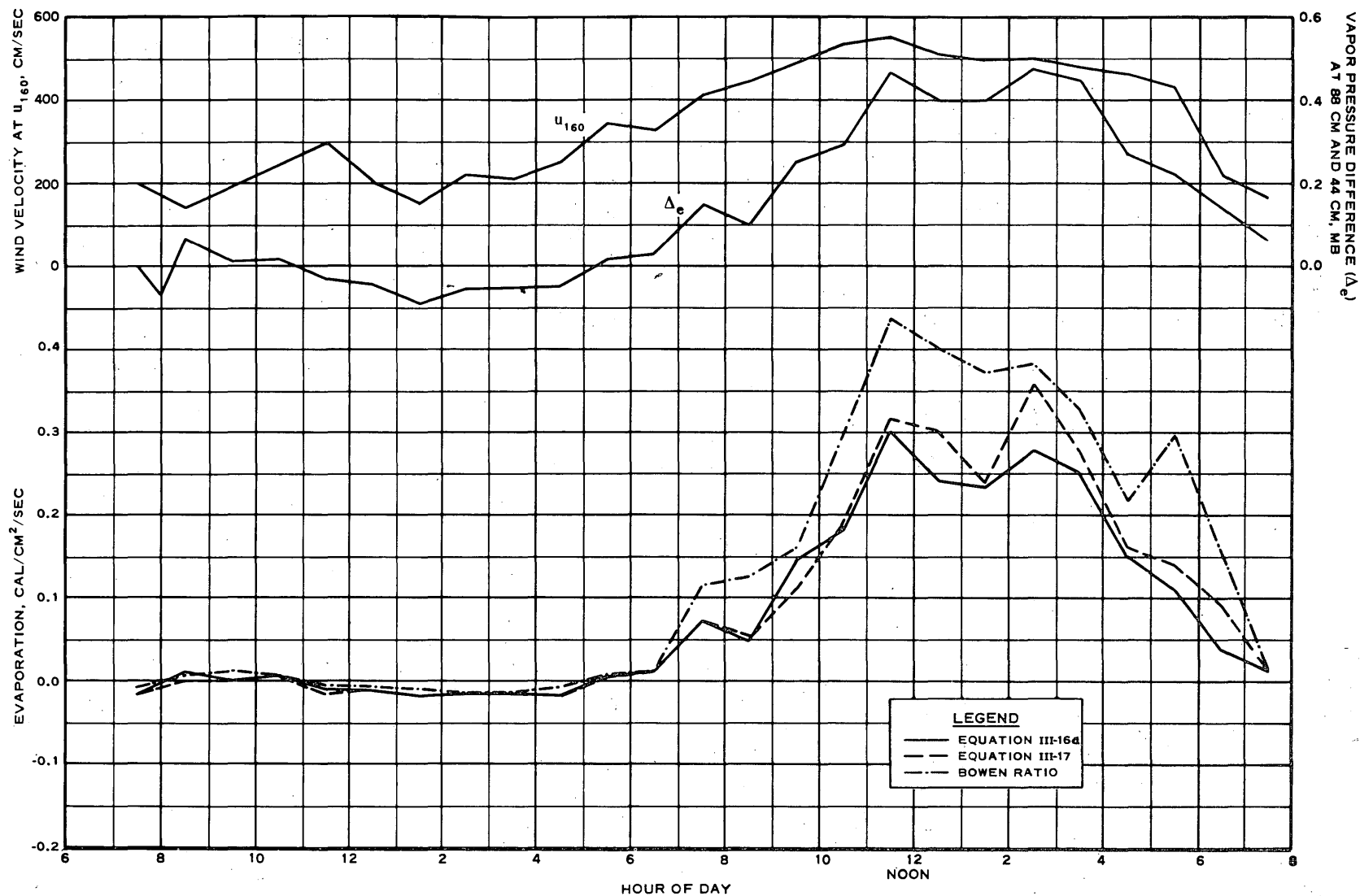


Figure III-7. Evaporation vs hour of day from various formulae. 18-19 August 1953, O'Neill, Nebraska

Combining K_v in equation III-17 with the vapor pressure difference gives column 9, the evaporation heat loss from equation III-17. Column 10 is taken from the UW report and represents their computed values of evaporative heat loss using the Bowen ratio.

The results from the three methods may be best compared by means of a plot of evaporation versus time which is shown in Figure III-7. On the same figure, plots of wind velocity at 160 cm and the differences between vapor pressures at the two heights of 44 and 88 cm are shown for comparison. It will be seen that the shapes of the wind velocity and vapor pressure difference curves are quite similar. This commonly occurs especially in the Great Plains data, and a like correspondence appears in a temperature difference plot as a periodic cycle of strong daytime and low nighttime winds and high and low gradients asserts itself.

It may be said here parenthetically that this is one of the major difficulties of obtaining analysable data. Practically all the days for which these complicated measurements have been made and for which data are published are typical days; any occasion on which strong nighttime or low daytime winds occur is very rare. Also, on no occasion are medium to strong winds recorded.

This is quite understandable from the point of view of obtaining reliable information, but it makes the task of compiling data in all types of conditions almost impossible. It is important to remember this point when extrapolations of the information are made to arctic or subarctic locations, as it may affect the results considerably.

Naturally, the vapor pressure difference curve indicates the direction of evaporation, that is, evaporation or condensation.

There is a fundamental difference between the curves obtained from a direct or indirect consideration of the eddy diffusion coefficient and those derived from the Bowen ratio. The Bowen ratio is applied to the difference between the radiative and soil heat fluxes to separate evaporative and convective flows. The amount of evaporation depends to a certain extent on the temperature gradient and convective heat flux. While these quantities are not implicit in the other calculations, they may have an explicit effect since it is unlikely that condensation will take place in a negative temperature gradient unless the surface of the soil is a deliquescent salt.

It would appear that equations III-16a and III-17 agree fairly closely on the amounts of evaporation occurring, and differ quite widely from the Bowen ratio method. It is probable from other considerations, taken later, that the latter calculation overestimates the amount of evaporative flux and underestimates the convective flux.

Since equation III-17 takes into account changes in stability and in the resultant velocity profile, it is likely that it gives a better value for the eddy diffusion coefficient than Holzman's formula based on a logarithmic distribution. However, it can be seen that the relatively simple calculations involved in equation III-16a give a reliable estimate of evaporation, if equation III-17 is considered to be correct. It will be observed that Holzman's equation underestimates the evaporation during unstable conditions during the day and gives a close identification during neutral equilibrium, for which it is intended. No reliable conclusions can be drawn from the amounts observed at night. These remarks are substantiated by the final report on the Lake Hefner studies, from which it can also be derived that equation III-16a overestimates evaporation during stable conditions.

A further check on the reliability of the equations can be obtained. During the Great Plains Turbulence Project studies, the University of California (hereafter referred to as UC) group, under J.E. Vehrencamp, made direct studies of the shearing stress at the surface of the earth by means of a so-called "shear meter." This meter consisted of a disk floating in a tank full of liquid, and fitting the tank quite closely. The surface of the disk was made closely similar in roughness characteristics to the natural surface of the ground, and the wind acting on this surface tended to displace it. By measuring the force required to restrain the disk, a continuous record of the wind shear stress at the ground surface was obtained. The results were published in a report¹ and took the form of a plot of shear stress versus wind velocity at 2 meters height.

Now in the derivation of Table III-3 the value of K_V was found, indirectly for equation III-16a and the Bowen ratio, and directly for equation III-17. As before, if it is postulated that $K_V = K_M$, it is possible to calculate the shear stress at the ground surface from these expressions. In the case of equation III-17 the friction velocity has already been found and tabulated, and it is a simple matter to evaluate the shear stress at ground surface from this expression.

For the other equations once K has been found, the expression

$$\tau = \rho K_M \frac{d\bar{u}}{dz} \quad (\text{III-3b})$$

can be used to find τ , the shear stress, which is assumed constant with height, at least over the first few meters above the earth's surface.

The results of these calculations are tabulated in Table III-4 to give a picture

Table III-4

Changing Shear Stress at Ground Surface During Day, O'Neill, Nebraska, 18-19 August 1953

No. of Observations	Time	u_{2m}	III-16 a	III-17	Bowen
			r_o	r_o	r_o
1	19-20	217	0.346	0.285	0.121
2	20-21	155	0.219	0.031	0.108
3	21-22	215	0.304	0.283	1.32
4	22-23	266	0.581	0.440	0.407
5	23-24	326	0.825	0.878	0.165
6	24-01	219	0.354	0.360	0.177
7	01-02	168	0.211	0.100	0.097
8	02-03	236	0.451	0.341	0.389
9	03-04	226	0.403	0.326	0.290
10	04-05	268	0.609	0.471	0.180
11	05-06	360	1.11	0.927	1.11
12	06-07	347	1.04	0.887	0.962
13	07-08	423	1.53	1.46	2.56
14	08-09	458	1.77	1.88	4.42
15	09-10	506	2.54	1.82	2.85
16	10-11	550	2.58	2.65	4.19
17	11-12	565	2.77	2.83	3.99
18	12-13	525	2.30	2.84	3.83
19	13-14	512	2.27	2.32	3.59
20	14-15	515	2.14	2.68	2.95
21	15-16	494	2.07	2.26	2.68
22	16-17	475	1.88	2.03	2.79
23	17-18	442	1.55	1.92	4.22
24	18-19	---	----	-----	-----
25	19-20	180	0.263	0.077	0.217

of the changing shear stress throughout the day. In order to compare them with the experimentally observed results of the UC group, they are also plotted in Figure III-8, together with a line A, representing the average of the experiments observed over all the periods of observation, and a line B representing the average of the

stresses observed during the 18-19 August period only. This latter line consistently differs from line A, and some comment is devoted to the fact in the UC report, although no conclusions are drawn. From this graph it is seen that equation III-17 seems to give an adequate description of the shearing stress, at least over the range noted. Therefore the value of K computed from it has significance for computations in this range. It is impossible to obtain any other check on the correctness of such equations since the fluxes of evaporation and convective heat cannot be measured directly (unless the moisture loss from the ground is measured) and only shear force lends itself to a determination.

Equation III-16a, although giving quite close correlation with the observed data at higher velocities, gives consistently too high shear stresses at the lower end of the scale. It gives good results considering the relatively small amount of work which went into the calculations based on it. The reason for the high stresses at the lower wind velocities lies in the equation's reliance on the logarithmic law of wind velocity distribution. It is not directly attributable only to the fact of the low velocities, but to their coincidence with stable conditions when the logarithmic law no longer holds. The Bowen ratio calculations give widely scattered points, showing the correct trend, but giving a most unreliable quantitative estimate of it. They appear to give values of shear stress which are too low at low velocities, and too high at higher, probably because of erroneous assumptions regarding heat flux under different stability conditions.

3-13. Variation of Eddy Diffusion Coefficient with Measurable Parameters. As it has been demonstrated that equation III-17 describes the effect of the eddy diffusion phenomenon in the range of wind velocities which has been considered at least adequately, it is desirable to prepare diagrams showing the influence of the surface roughness on the wind velocity at a fixed level and the stability of the atmosphere on the coefficient of eddy diffusion. The coefficient can then be applied to an observed gradient to determine either the evaporation or the shear stress across a horizontal surface.

To determine an eddy diffusion coefficient for a particular level (it varies with height) in the atmosphere, it will be necessary to know the value of z_0 , the surface roughness parameter, the wind velocity at some fixed reference height, such as the U.S. Weather Bureau standard of 4-1/2 ft for temperature observations, and the stability of the atmosphere, expressed as a Richardson number. It is realized that in many cases, particularly near airports, the wind velocity is not measured at 4-1/2 ft

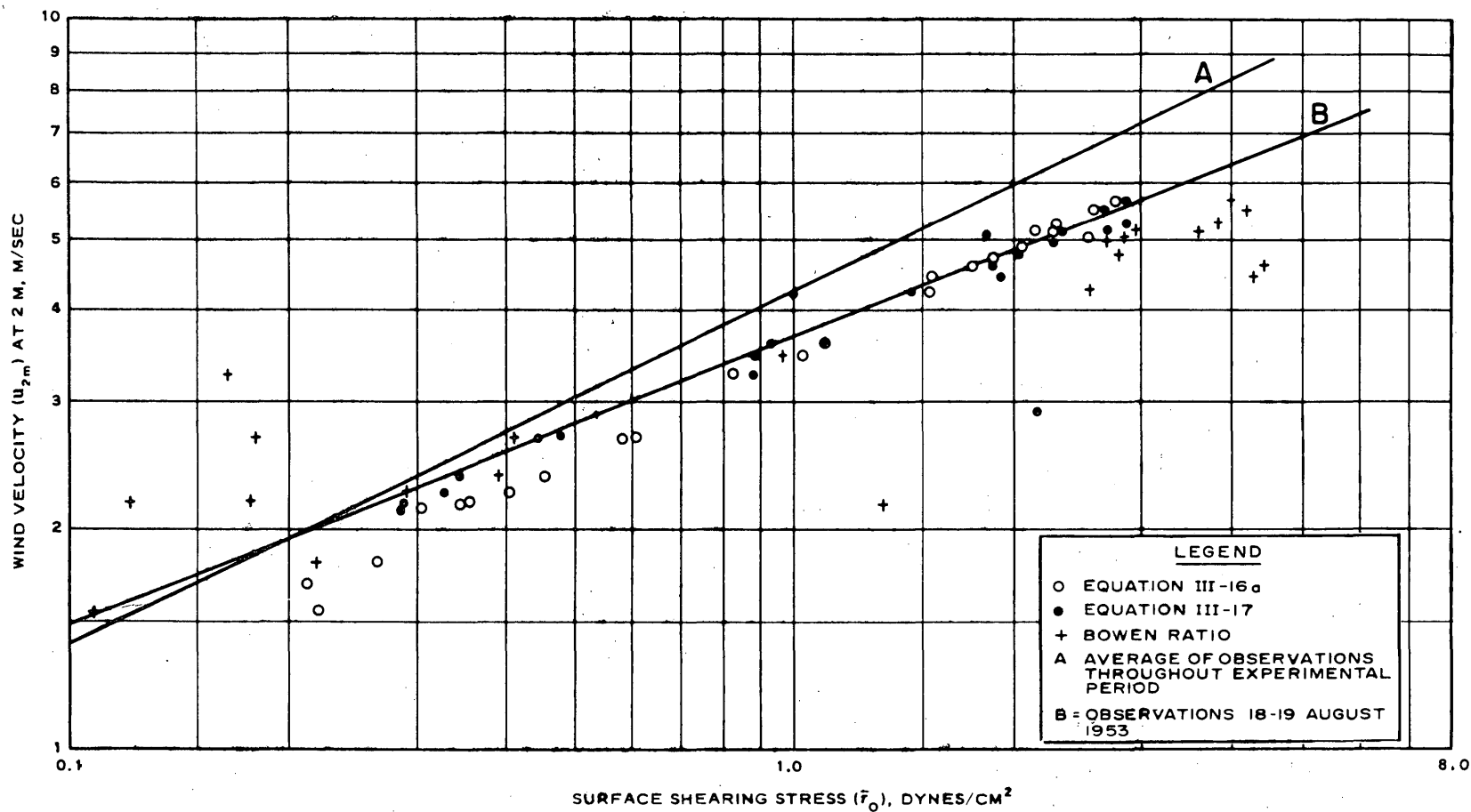


Figure III-8. Wind velocity at 2 m vs surface shearing stress. Calculated and observed 18-19 August 1953, O'Neill, Nebraska

but at much higher levels, nearer 50 ft, and it may be necessary for future calculations to relate the tables to the wind velocity at this height or to provide means of reducing the velocity to the corresponding one holding at the lower level.

Table III-5, which is taken from Deacon²⁹ and checked by calculations made on data from Seabrook Farms, New Jersey, and O'Neill, Nebraska, indicate values of z_0

Table III-5

Table of Roughness Parameter, z_0 cm, for Typical Surfaces
Limiting velocity given is that above which rough flow occurs
according to Schlichting⁹⁶

z_0	Surface	Lower Limiting Velocity cm/sec	Values from Analyses Herein
20			
10		0.25	
8	Turnip field		
6			
4	Wheat fallow land		
2	Grass airport		Prairie surface, O'Neill, Nebraska, 18-19 August 1953
1.0		4.5	
0.8	Mown grass		
0.6			
0.4			
0.2	Baseball Cricket field		
0.1			
0.08	Natural snow	65	
0.06	Sun-baked sandy alluvium		5- to 6-mm-high spinach Seabrook, N.J., 4-5 September 1952
0.04			
0.02			Bare soil surface Seabrook, N.J., 21 August 1952
0.01	Smooth snow on prepared short grass	85	
0.008			
0.006	Most of these results are		
0.004	probably incorrect since		
0.002	Re is likely to be $\ll 10^9$		
0.001			

for different types of surface. The wind velocity at any location can be obtained from weather bureau data, and, if necessary, interpolated to the value holding at the point of interest. Finally, as it is unlikely that sufficient information will ever be available to enable calculations of the Richardson number to be carried out, it will probably be sufficient only to note either the time of day or whether it is day or night when the value must be calculated. Weekly or longer term averages will be considered later.

By inserting different plausible values of wind velocity at a particular reference height, the roughness parameter, and β in equation III-12 it is possible to plot curves of the eddy diffusion coefficient against all these variables. This has been done and these curves will be found plotted in Figures III-9a to d. Deacon's empirical curve of β versus Ri and other material analysed for this work were used to obtain a reasonable range of β , itself a stability parameter, for the calculations.

In equilibrium conditions Deacon's formula corresponds to the logarithmic law, equation III-6b, and this law has been plotted for such conditions on the figures. Also on the figures is shown the curve of smooth surface adiabatic flow, equation III-5. This raises a further problem: at some small range of velocities there is, for any given roughness of ground surface, a transition from smooth to rough surface flow, considering neutral equilibrium conditions for simplicity. The only way this transition can be found is by investigating actual flows in nature to see if the shear stress at the ground surface can be calculated from the smooth or rough-surface flow equation. An attempt has been made to do this, but the data are very limited. Not enough points can be obtained to point out definite trends for any one degree of surface roughness. The best that can be done is to indicate the direction of the trend which cannot be very far wrong, and in which small errors will not make much difference for calculations in which estimated soil properties must also be utilized.

There is a further indication of the velocity range below which smooth surface flow occurs, and that is by calculation of the thickness of the boundary layer. A method of evaluating this has been worked out, and is mentioned later. When the results were applied to a surface roughness parameter of 0.017 cm, it appeared that even quite low wind velocities (about 100 cm/sec) were still in a transitional zone between the two flows. Again it is unfortunate that no data are available on velocities and temperatures above airfield pavements which are probably much smoother even than this.

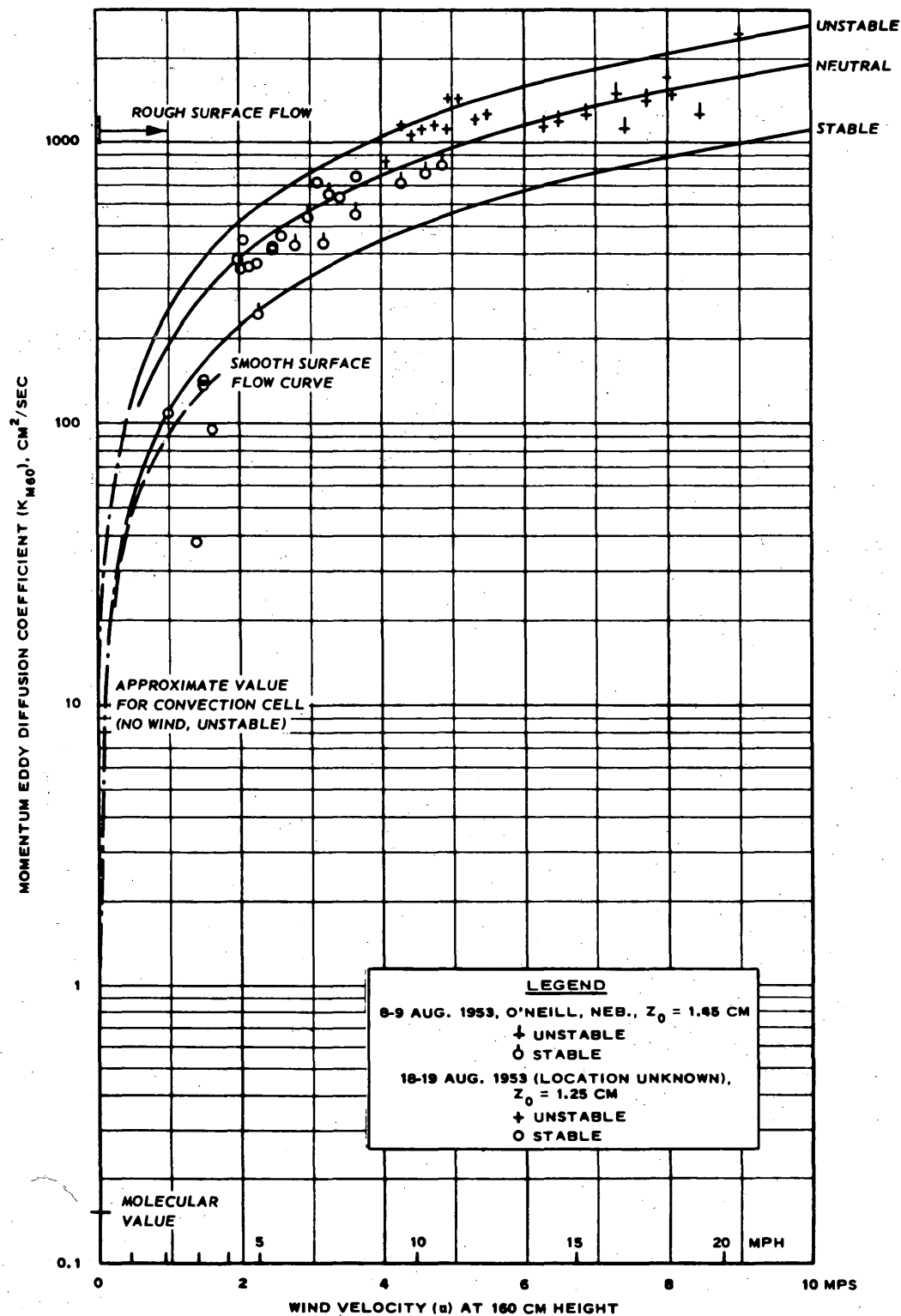


Figure III-9a. Eddy diffusion coefficient vs wind velocity at 160-cm height at different stabilities and surface roughnesses, $z_0 = 1.0$ cm

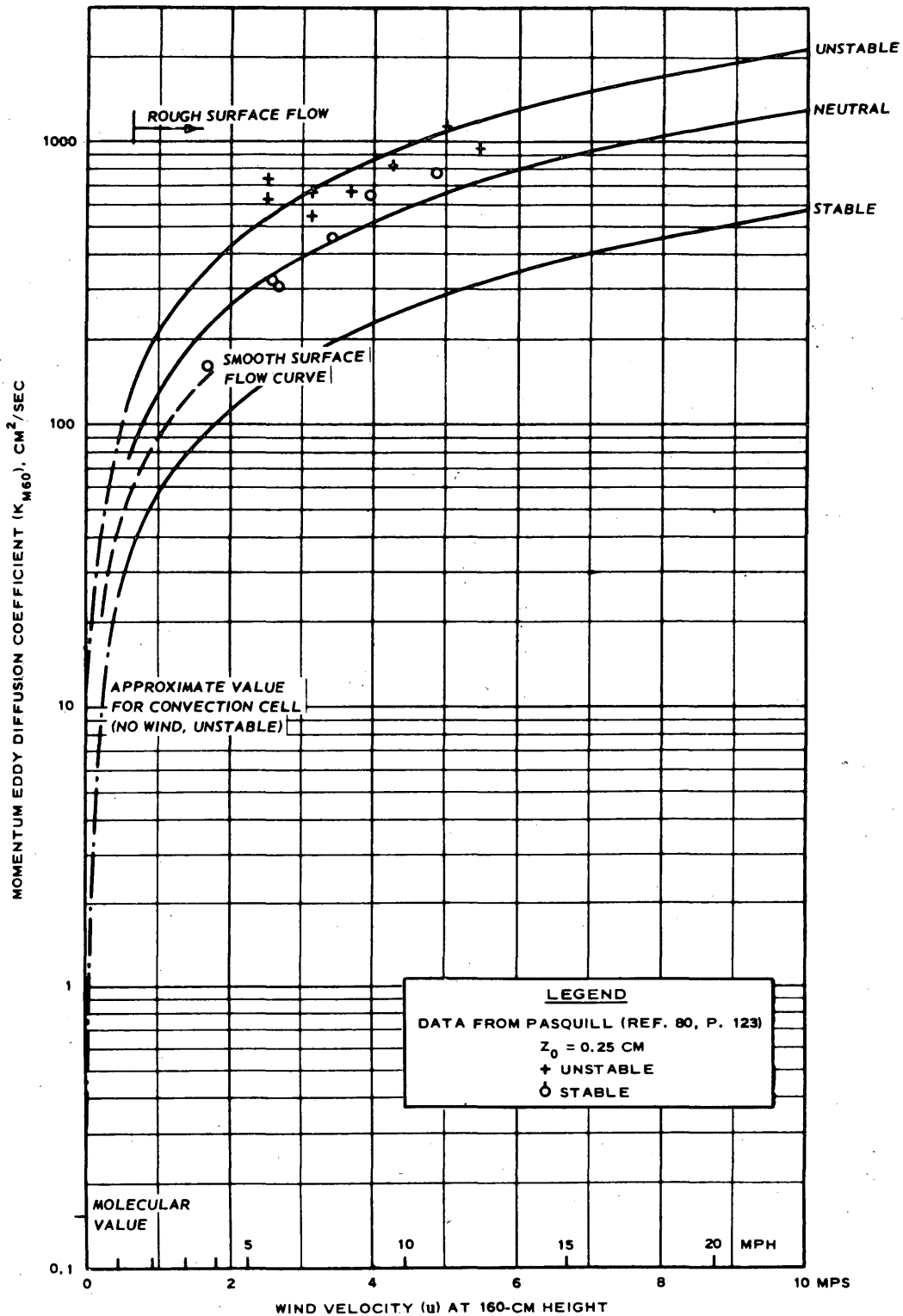


Figure III-9b. Eddy diffusion coefficient vs wind velocity at 160-cm height at different stabilities and surface roughnesses, $z_0 = 0.1$ cm

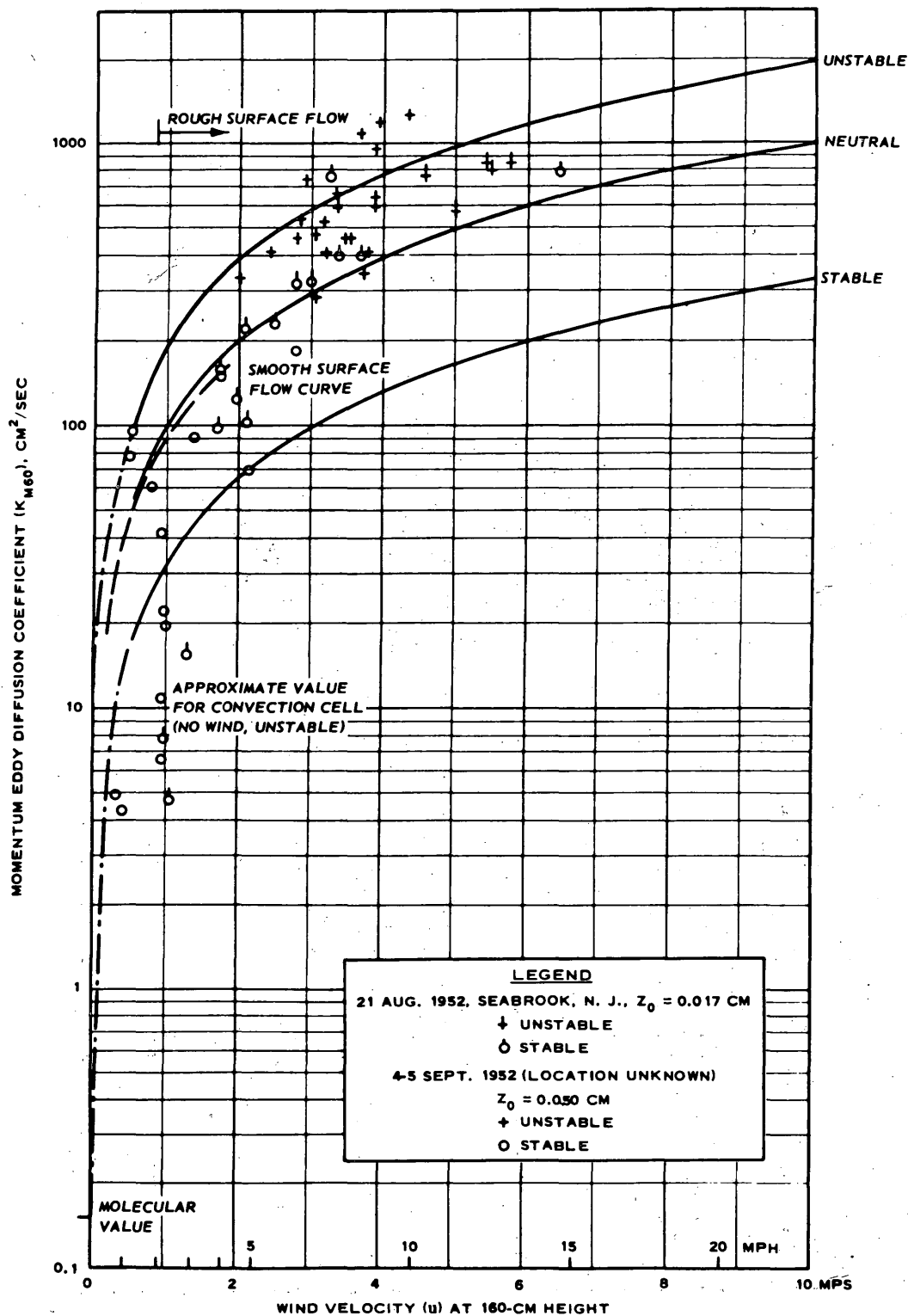


Figure III-9c. Eddy diffusion coefficient vs wind velocity at 160-cm height at different stabilities and surface roughnesses, $z_0 = 0.01$ cm

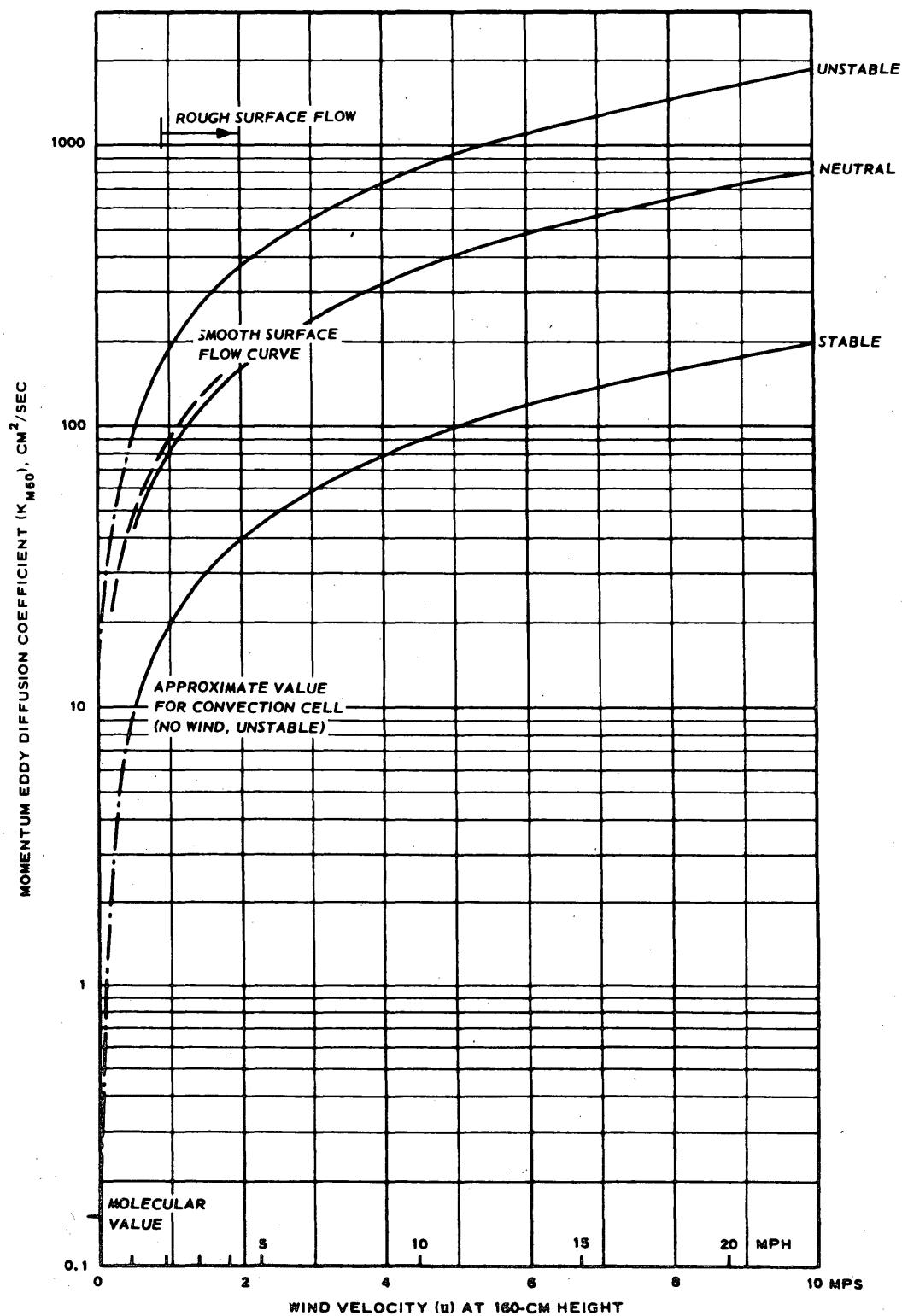


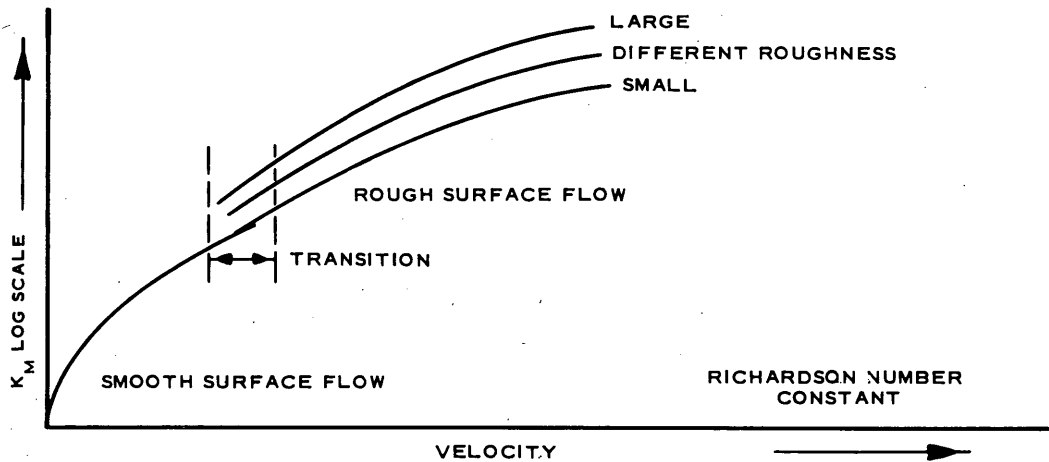
Figure III-9d. Eddy diffusion coefficient vs wind velocity at 160-cm height at different stabilities and surface roughnesses, $z_0 = 0.001$ cm

3-14. Stable Temperature Stratification. So far only unstable and equilibrium conditions have been particularly dealt with. There is the further situation of stable temperature stratification in the atmosphere near the ground, which occurs when the surface is colder than the air, predominantly during the night. In this case the air nearest the ground is colder and heavier than the air above and consequently resists any attempts by turbulence to displace it to higher levels. Thus it has a damping effect on the turbulence, and it may be postulated that for given conditions of wind velocity at a certain level and a given roughness of ground, there will be a certain degree of stability or value of the Richardson number at which turbulence is restrained or damped out.

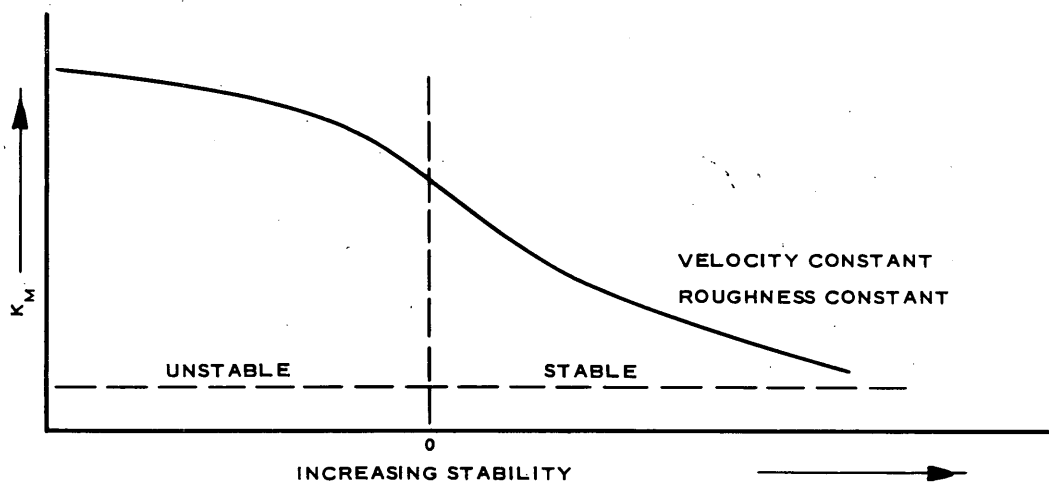
Richardson, Schlichting, and Taylor¹⁰³ have all examined the conditions under which turbulence can or cannot be maintained, but reach no agreement on the matter. Consequently it is now considered that there is probably no one value of the Richardson number which is critical for the damping of turbulence. Indeed, Businger²¹ has shown that the value can vary considerably under different circumstances, as it depends on the amount of damping which has taken place. It is to be expected then, that at any particular roughness, for one velocity at a fixed level, there will be a spectrum of eddy diffusion coefficients, varying from high values, indicating high turbulence, during very unstable conditions, to low values (which may for low velocities become of order very close to the molecular diffusivity) at stable times, and ranging about the value represented by neutral equilibrium conditions.

As the stability increases, the turbulence will be indicated by an increasing velocity gradient with respect to height, until it may finally be represented by the laminar flow velocity gradient, r/μ . Now if, for one particular roughness, a range of velocities is considered, it will obviously be true that at a low velocity the range of degree of turbulence for different stabilities is relatively larger than it will be at a higher velocity. In other words, convective forces predominate at low wind velocities, and temperature stratification plays only a small part in heat transfer at higher velocities. These ideas are shown qualitatively in the diagrams Figure III-10 a, b, and c.

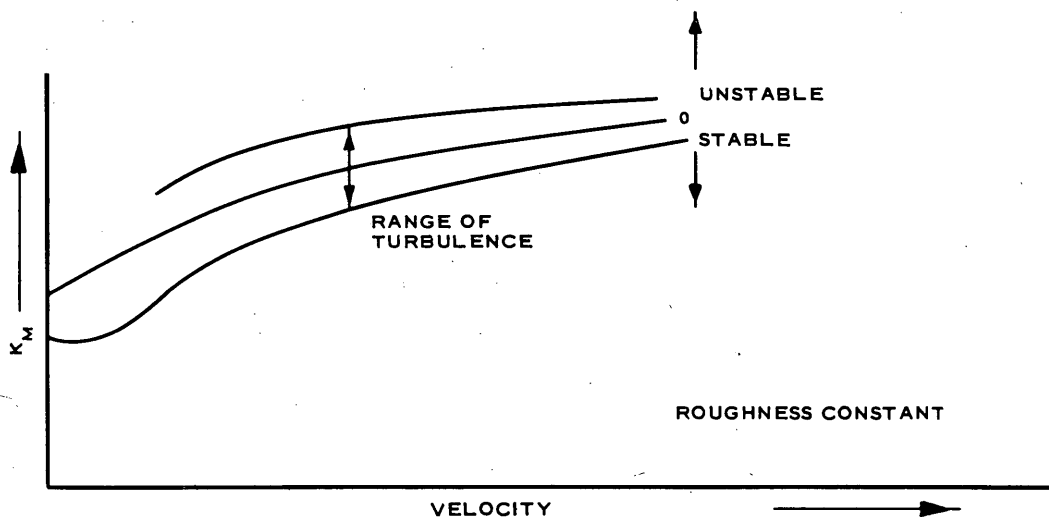
Then, on the basis of the curves shown in Figures III-9a to d it becomes possible to predict either the shear stress at the earth's surface under given conditions, or the amount of evaporation, provided the vapor pressure gradient in the appropriate height interval is known. However, for the purposes of airfield pavement studies, it



a. Varying roughness and velocity



b. Varying stability



c. Varying velocity

Figure III-10. Eddy diffusion coefficient (K_M) vs various factors

is of primary interest to calculate the heat transfer to the air by convective and mechanical turbulence, not by evaporation.

3-15. Difference Between K_H and K_V . It is therefore of interest to study the relation between the eddy diffusion coefficient of heat transfer, and that for vapor or momentum transfer. It has already been stated that

$$K_H = K_M, K_V$$

on the basis of theoretical^{35,86} and experimental studies,^{106,80,116} although this has been disputed by Robinson and Rider.⁹⁰ Partly in the hope of adding some more information to the scanty data already existing, and partly to utilize data which seem to satisfy more rigorous requirements than previous work, analyses were made of the Great Plains Turbulence Project reports once more.

In the case of this work, it was assumed from previous studies that $K_M = K_V$, and that a determination of K_M obtained from equation III-17 and which could be applied to the vapor pressure gradient over the same height interval as that to which K_V referred would give a reasonably correct estimate of the heat loss to the atmosphere by evaporation. As in the ground surface heat balance equation, the net radiation, ground heat flow, and evaporative terms were known, the remaining heat quantity must be that transferred to the air. If the temperature potential gradient between the same two levels as before was calculated and used in the equation

$$q_{co} = \rho c_p K_H \left(\frac{\partial T}{\partial z} + \Gamma \right) \quad (\text{III-24})$$

which is of a form similar to the shear stress equation III-3b, it will be possible to calculate a value of K_H . The most convenient way of examining the value of K_H obtained is to compare it with the calculated value of K_M .

Just as in all the other factors considered, it may be assumed that if K_H is not equal to K_M , then the amount of inequality will depend on the wind velocity, the roughness, and the temperature gradient in the layer. As both the roughness and the wind velocity affect, and are, in a sense, included in the velocity gradient du/dz , and since the Richardson number includes both the temperature and velocity gradient, it is considered suitable to attempt to correlate the ratio K_H/K_M with the Richardson number.

The data analysed previously (18-19 August 1953, O'Neill, Nebraska) have been carried a little further to compute the value of K_H/K_M , as is shown in Table III-6. The resulting plot is Figure III-11. Some data of Pasquill's are also germane to the problem; they have been shown in Table III-6, and some points on the Figure III-11 are the result of this reference. In Table III-6, the calculated value of the heat loss due to evaporation, from Deacon's equation, has been subtracted from the heat loss due to evaporation and convection together, to give the convection amount, and then, by applying the temperature gradient observed, values of K_H have been computed.

Vehrencamp in his 1951 report¹¹⁶ made a graph of K_H/K_M versus time of day which has been reproduced as Figure III-12. Although no figures have been given with which to estimate the Richardson number, it can be seen that the results are in substantial agreement with Figure III-11, when it is remembered that the daytime values represent relatively high instabilities, and the evening values tend toward neutral or stable conditions.

On careful consideration, it was felt that any further analysis of the variation of K_H/K_M as a function of stability would not advance the problem from the point of view of programming the analog computer, and it was decided to approximate the results of Figure III-11 by means of the lines shown on it. This makes use of the assumption that under neutral and stable conditions $K_H = K_M$, K_V . This seems to be substantiated both by Pasquill's and Vehrencamp's data, although Pasquill does not think it proved, and that in unstable conditions K_H tends towards a value of 2-1/2 to 3 times K_M . A further simplification could be introduced for long-term estimations of K_H , by assuming that in the daytime, when there is a considerable amount of net radiation into the ground surface, $K_H/K_M = 2$, and at night equals unity.

3-16. Overall Heat-Transfer Coefficient. In order to utilize the analog computer, which is constructed on the basis of surface temperature variations, it is necessary that the data be set up in the form of an air temperature at a known elevation (which will be given) and a heat-transfer coefficient between that elevation and the ground surface. In this way there will be a flow of heat depending on the (potential) temperature gradient between the air and the ground surface and the transfer coefficient involved. So far it has been indicated how the eddy diffusion coefficient is found between two fixed levels above the soil surface, and it is necessary to find the relation between this coefficient and the one which applies to the air between ground surface and one fixed point in the air at which temperature is measured.

Table III-6
Ratio of K_H to K_V

For O'Neill, Nebraska, 18-19 August 1953

No. of Observations	q_{co}	K_H	$\frac{K_H}{K_V}$
1	+ 0.009	66	0.185
2	+ 0.019	101	2.650
3	+ 0.024	153	0.397
4	+ 0.050	350	0.850
5	+ 0.005	43	0.059
6	+ 0.014	125	0.277
7	+ 0.011	98	0.682
8	+ 0.023	282	0.777
9	+ 0.018	239	0.652
10	+ 0.003	45	0.105
11	+ 0.044	638	0.995
12	+ 0.029	548	0.870
13	-0.128	2190	2.510
14	-0.320	2567	2.395
15	-0.230	1327	1.584
16	-0.370	2243	1.833
17	-0.458	1945	1.536
18	-0.462	2000	1.407
19	-0.389	2172	1.896
20	-0.282	1370	0.953
21	-0.215	1470	1.252
22	-0.169	1820	1.633
23	-0.158		
24	+0.048	555	0.457
25	-0.035	158	1.640

From Pasquill's Data

Time	Date	K_{H75}	K_{V75}	R1	$\frac{K_H}{K_V}$
1030	10 March 1948	2150	1030	-27	2.09
1230		1820	840	-50	2.17
1530		----	950	0	
1730		430	370	+64	1.16
1930		690	570	+33	1.21
0940	24 March 1948	1270	780	-66	1.63
1130		1680	930	-125	1.81
1430		1650	830	-52	1.99
1630		----	810	-1	
1830		420	390	+58	1.08
1030	25 March 1948	2140	690	-51	3.10
1030	28 March 1948	1890	1400	-19	1.35
1230		2020	1180	-17	1.71
1900		240	190	+123	1.27

Figure III-11. Ratio of heat eddy diffusion coefficient to momentum eddy diffusion coefficient $\left(\frac{K_M}{K_{V,M}}\right)$ vs Richardson number

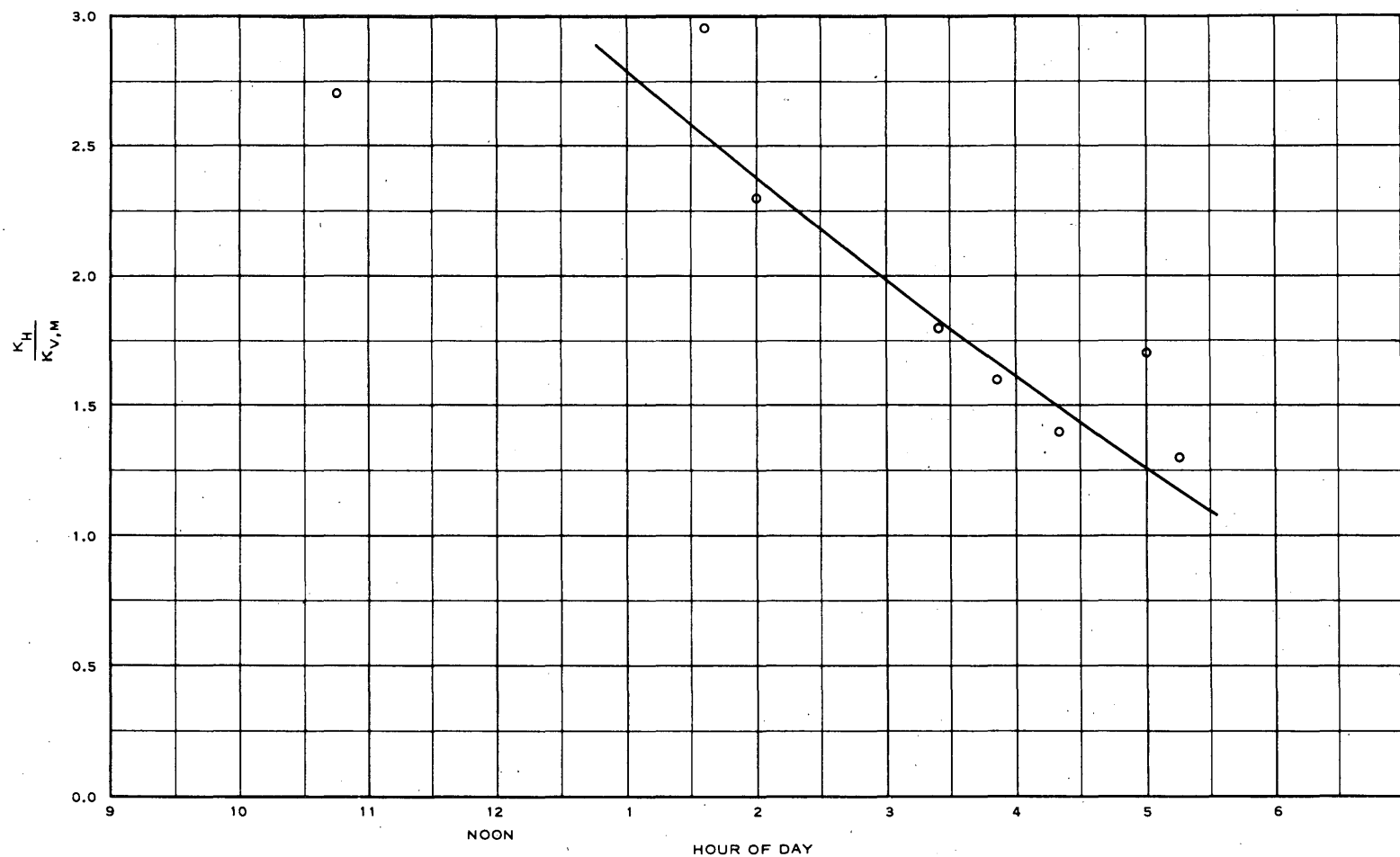


Figure III-12. Ratio of heat eddy diffusion coefficient to momentum eddy diffusion coefficient $\left(\frac{K_H}{K_{V,M}}\right)$ vs hour of day. Data from Vehrencamp (Ref. 116, p. 126)

It has been pointed out previously that the flux of heat over a particular period depends on the average eddy coefficient of heat transfer and the temperature gradient for the period considered and it therefore follows that for a known heat flux, the ratio of average eddy coefficients over two different zones will vary as the temperature gradients across the zones. In symbols this becomes

$$\frac{\kappa_{H1}}{\kappa_{H2}} = \frac{(\Delta T / \Delta z)_2}{(\Delta T / \Delta z)_1} \quad (\text{III-25})$$

It is necessary to choose a height at which the temperature and wind velocity data will be measured, and as the U.S. Weather Bureau readings are usually made in shelters between 4 and 6 ft above the ground surface, and since the UW and JH publications both give information taken at 160 cm or 5 ft 3 in., it was decided to refer all data to the 160-cm level. Other heights would involve probably unnecessary interpolation. Thus it is desired to obtain the temperature gradient between ground level and 160 cm, and to compare it with that existing between 40 and 80 cm. Unfortunately, in the Great Plains Project publications no data on ground surface temperatures are given and in only a few days at the JH organization at Seabrook, New Jersey, were they taken. Even then, because of the difficulty of obtaining representative surface temperatures, the data may not be very reliable. The latter temperature data were used for calculation and are shown in Table III-7, together with Figure III-13, but these may not constitute an adequate number of points.

Table III-7
Comparison of Temperature Gradients over Intervals 0-160 and 40-80 cm

23 July 1952, Seabrook, N. J.			21 August 1952, Seabrook, N. J.			15 April 1953, Seabrook, N. J.			23 April 1953, Seabrook, N. J.		
Time	ΔT	ΔT	Time	ΔT	ΔT	Time	ΔT	ΔT	Time	ΔT	ΔT
	Δz_{0-160}	Δz_{40-80}		Δz_{0-160}	Δz_{40-80}		Δz_{0-160}	Δz_{40-80}		Δz_{0-160}	Δz_{40-80}
06-07	-0.00913	+0.0013	00-01	+0.001437	+0.00165	08-09	-0.00641	-0.0015	07-08	+0.0185	+0.0095
07-08	-0.0235	-0.0048	01-02	+0.00125	+0.00165	09-10	-0.01561	-0.0035	08-09	+0.0168	+0.0048
08-09	-0.01712	-0.0045	02-03	-0.000125	+0.00140	10-11	-0.0283	-0.0060	09-10	-----	-----
09-10	-0.0267	-0.0043	03-04	+0.001563	+0.00015	11-12	-0.0299	-0.0068	10-11	+0.0038	+0.0025
10-11	-0.0384	-0.0075	04-05	+0.00524	+0.00165	12-13	-0.0239	-0.0055	11-12	+0.0065	+0.0010
11-12	-0.0441	-0.0100	05-06	+0.00319	+0.00165	13-14	-0.0235	-0.0048	12-13	-0.0026	-0.0010
12-13	-0.0391	-0.0118	06-07	-0.00231	-0.00035	14-15	-0.0145	-0.0038	13-14	+0.0140	+0.0020
13-14	-0.0456	-0.0138	07-08	-0.00556	-0.00185	15-16	-0.0169	-0.0030	14-15	+0.0062	+0.0028
14-15	-0.0494	-0.0103	08-09	-0.01662	-0.00385	16-17	+0.0026	+0.0008	15-16	+0.0153	+0.0055
15-16	-0.0224	-0.0058	09-10	-0.0235	-0.00310	17-18	+0.0081	+0.0040	16-17	+0.0086	+0.0038
16-17	-0.00144	-0.0010	10-11	-0.01324	-0.00410	18-19	+0.0103	+0.0043	17-18	+0.0126	+0.0060
17-18	-0.00225	-0.0010	11-12	-0.01181	-0.0021	19-20	+0.0100	+0.0043	18-19	+0.0327	+0.0140
18-19	+0.00269	+0.0002	12-13	-0.01704	-0.00435	20-21	+0.0074	+0.0020	19-20	+0.0259	+0.0170
			13-14	-0.01082	-0.00335	21-22	+0.0114	+0.0033	20-21	+0.0256	+0.0178
			14-15	-----	-----	22-23	+0.0136	+0.0030	22-23	+0.0243	+0.0115
			15-16	-0.0248	-0.00535	23-24	+0.0096	+0.0025	23-24	+0.0395	+0.0258
			16-17	-0.00281	-0.00185	24-01	+0.0129	+0.0040			
			17-18	-0.00243	-0.0006	01-02	+0.0136	+0.0048			
			18-19	+0.00387	+0.0004						
			19-20	+0.00256	+0.0004						
			20-21	+0.00462	+0.00065						
			21-22	+0.00699	+0.0014						
			22-23	+0.00293	+0.0004						
			23-24	+0.00331	+0.00065						

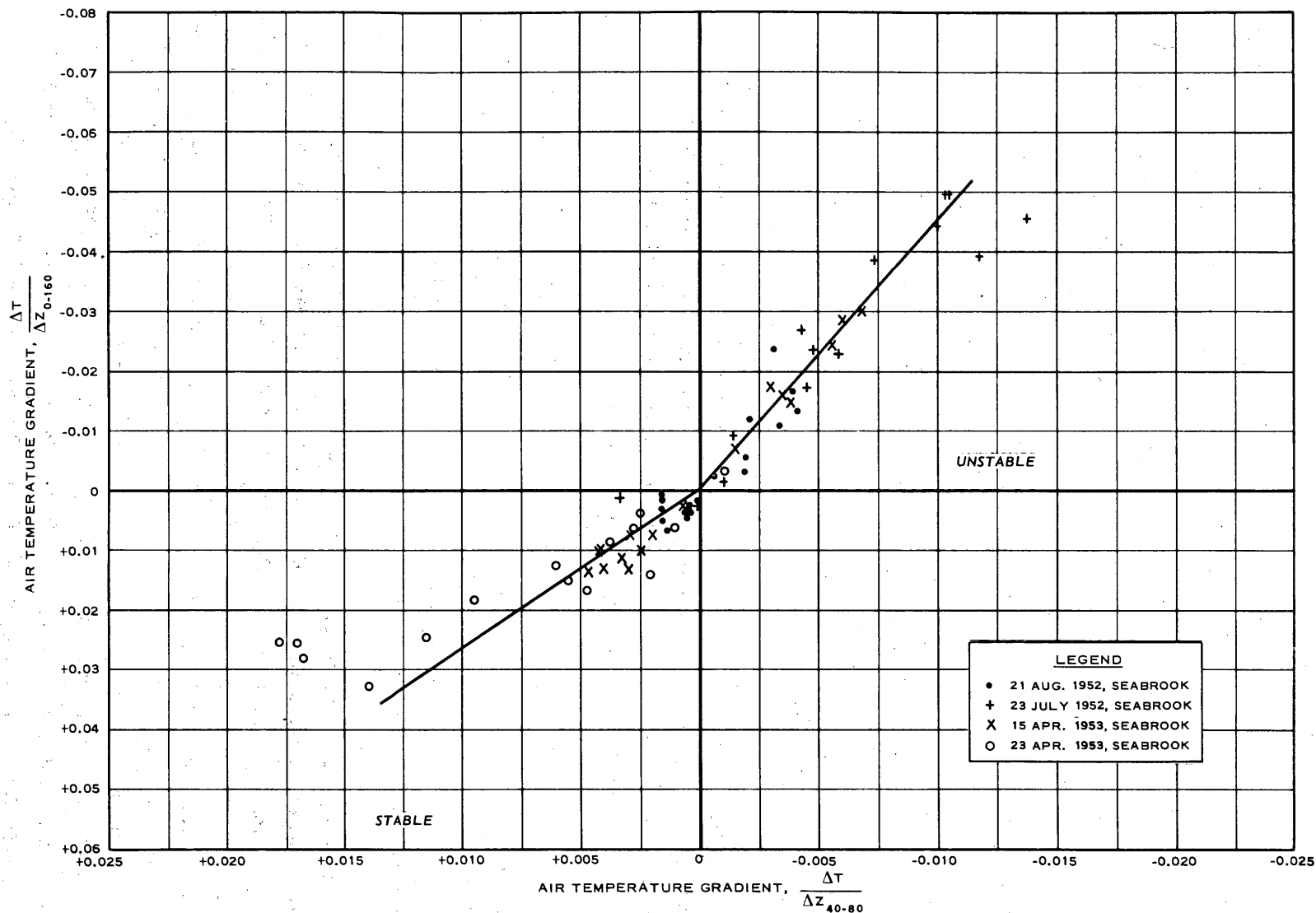


Figure III-13. Air temperature gradients, 0-160 cm vs 40-80 cm

It is possible to obtain the ground surface temperature by calculation, providing the temperature at some depth in the ground is known as a function of time, and providing the thermal properties of the soil between the surface and the depth considered are known. The method will be given here, although a much fuller description of it will be demonstrated in Part V where the ground surface temperature is examined in greater detail.

According to Jakob,⁵⁴ (p 293) if the surface temperature of a semi-infinite homogeneous solid oscillates according to the function

$$T_{s,t} - T_M = -P_1 \cos\left(\frac{\pi t}{12} - \epsilon_1\right) \quad (\text{III-26})$$

the temperature at a depth z and time t is given by

$$T_{z,t} - (T_M + a z) = -P_1 e^{-mz} \cos\left[\frac{\pi t}{12} - (\epsilon_1 + mz)\right] \quad (\text{III-27})$$

where $m = (\pi/24\alpha)^{1/2}$ and t is measured from midnight.

It is noted that the temperature in each case is represented as oscillating about a mean which varies with depth, the variation being taken linearly, as it has been observed in the temperature data that the mean temperature varies at different depths over one day, in a manner depending on the season of the year.

If an experimentally observed temperature curve at some depth z is fitted with a cosine curve of the form III-27 using the known soil thermal constants, it is possible to match the constants from the experimental curve with those in the equation III-27 to find values for P_1 and ϵ_1 . Substitution of these values in equation III-26 gives the temperature variation over the day at the surface.

Generally speaking, the actual temperature at the surface fluctuates considerably on days with cloud or variable wind, and the soil acts as a thermal filter so that the extreme fluctuations are smoothed out. Even so, at depths of a few centimeters, it may be difficult to fit a cosine curve well enough. If greater depths are chosen so that the fluctuations have been eliminated, the calculated surface temperature will be inaccurate and the computed temperature variations will not reflect the true oscillations. It is therefore necessary to choose days when the surface minor fluctuations will be at a minimum by examining the soil and air temperature data carefully, in other words, days of steady wind and little or no cloud. The derivation of the soil thermal properties is a major task in itself, it being very difficult to obtain them accurately. It is

possible to do as Lettau⁶⁶ does and carry out a Fourier Series "synthesis" on the curves of temperature versus depth, which will yield the most accurate values of the diffusivity coefficient of the soil and its variation with time and depth, but this method is difficult and time consuming, rendering it inconvenient for use here. It is possible, however, to arrive at a good estimate of the soil properties between two depths by proceeding with a method stated in the above paragraphs for finding the surface temperature. In it equation III-27 is fitted to the curves of temperature versus time for two known depths; the value of m , involving the thermal diffusivity of the soil, is unknown. The equation coefficients are equated to determine m , as follows:

$$\text{At depth } z_1, \quad P_1 e^{-m z_1} = C_1 \quad (\text{III-28a})$$

$$\text{At depth } z_2, \quad P_1 e^{-m z_2} = C_2 \quad (\text{III-28b})$$

where C_n is the half amplitude of the temperature wave at depth z_n .

Divide equation III-28a by III-28b to get

$$e^{m(z_2 - z_1)} = C_1/C_2 \quad (\text{III-29})$$

if $z_2 > z_1$, by which m can be calculated. When this is done for two or three different depths, it will be found that m varies, due to the changing properties of the soil, changing most near the surface. As the depth nearest the surface at which measurements usually are taken is 2.5 cm, the value of m can be determined only as an average between 2.5 cm and 5 cm. If the 5-cm depth temperature-time curve is to be used to find the surface temperature, this value of m may not hold over the whole 5 cm. Because of the variation which takes place in the upper 2.5 cm, errors may be large. In practice, it was not found possible to compute the surface temperatures accurately enough for the purpose of assisting the plotting of Figure III-13. The data considered for the study were taken from the O'Neill, Nebraska, project, in which the soil properties were obtained by means of weighings and water-content determinations. Compared with calculations from equations III-28 and III-29, these properties show a large discrepancy.

Although information is scarce, the data plotted on Figure III-13 are sufficient to show a trend and may therefore be discussed on a tentative basis. Some care must be taken in the evaluation of the data. If equation III-15,

$$K_M = k_O z_O u_* \left(\frac{z}{z_O} \right)^\beta \quad (\text{III-15})$$

is examined it will be noted that, provided rough surface flow is taking place, u_* is constant with height (that is to say, in the expression for u_* , substitution of a different velocity at a different height results in the same value of u_*), k_O , z_O are constant, and therefore K_M varies directly as z to the power β . Since in the data and intervals being considered, β does not vary very much from unity, and since β more nearly approaches unity as the ground surface is approached, it can be said that K_M is approximately proportional to height directly.

If two heights are taken close together, a value for K_M can be calculated from the wind velocity information over a height interval involved. This can be done to obtain another K_M at a different height interval. It would be expected that these two K_M 's would bear a relation to one another as did the average height intervals they represented. It is for this reason that the K_M value computed over the range between 40 and 80 cm is termed the K_M holding at 60 cm or K_{M60} . In spite of a variation of Richardson's number with height it would also be expected that the corresponding values of K_H calculated from those K_M 's would also be roughly proportional to their heights, or, in other words, using the constancy of the heat flux through all horizontal reference planes, the value of the temperature gradient varies as the height directly. This will rarely be exact, especially if two considerably different heights are used.

The above statements are approximately true only if the measurements and calculations are made in the turbulent zone, because Deacon's equation holds for rough surface flow, and describes conditions down to the boundary layer. It can therefore be used only above the upper boundary of laminar flow. Consequently, in plotting $(\Delta T / \Delta z)_{20-30}$ versus $(\Delta T / \Delta z)_{40-50}$ it would be reasonable to expect a linear relationship over the greater portion of the data.

In the example which is being considered, it must be noted that the temperature difference in one case is between the level of 0 and 160 cm, and in the other, between 40 and 80 cm, and therefore linearity would not be anticipated. The overall coefficient of heat transfer between 0 and 160 cm involves flow of heat through both a laminar layer and a turbulent zone (neglecting a transition which must also exist) in which two different laws hold. The zones represent two widely different values of transfer coefficient. On the other hand, flow between 40 and 80 cm is certainly turbulent, and one eddy diffusion coefficient may be applied to its properties to determine a heat flow or a shear stress.

The discussion so far is entirely a qualitative one, and it must be remembered that the statements made are only as true as the application of present theory to atmospheric conditions permits.

However, perhaps some quantitative information may be obtained by an analysis of the heat-transfer situation, considered on one side from the point of view of heat flow between ground surface and a height of z_1 , and on the other from that of flow across the zone between z_2 and z_3 , as in Figure III-14a and b. In Figure III-14a the laminar layer is considered to be distinctly bounded with molecular diffusion of properties below the boundary and turbulent or eddy diffusion above.

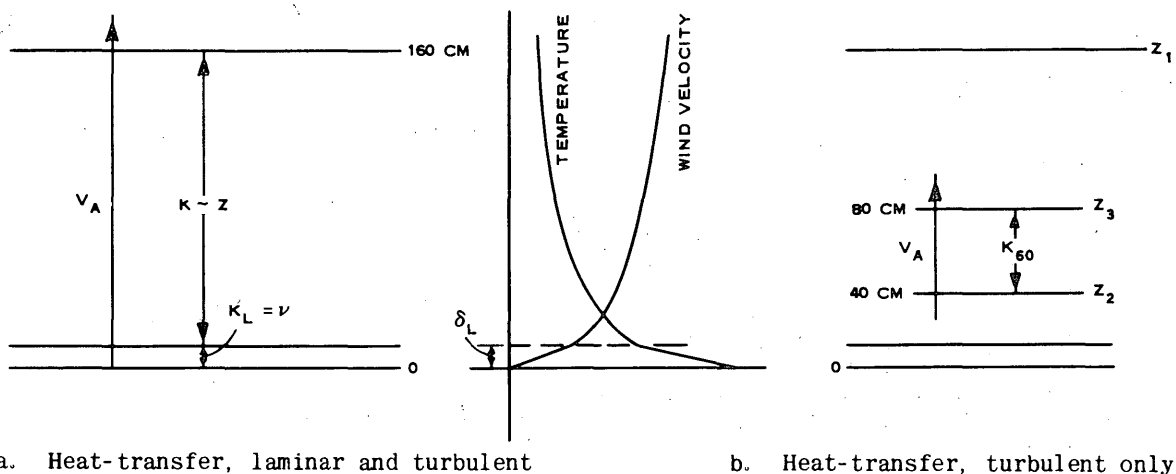


Figure III-14. Conditions prevailing in the 0- to 160-cm zone above the earth's surface

Then, for the whole height $0 - z_1$, the equation,

$$q_c = \rho c_p \frac{K_{Ht}}{z_1} (T_s - T_{z_1}) \quad (\text{III-30})$$

in which K_{Ht} is a conductivity applying over the whole range, can be written.

The resistances, reciprocals of conductances, for both the laminar and turbulent zone vary,

$$\frac{z_1}{K_{Ht}} = \frac{\delta_H}{K_\delta} + \frac{z_1 - \delta_H}{K_H (\delta_H - z_1)} \quad (\text{III-31})$$

If it is considered that, in the range δ_H to z_1 , K_M is directly proportional to height, then the average value equal to that obtaining at the height $(z_1/2) + (\delta_H/2)$, or, approximately, $z_1/2$ can be substituted. It can also be said that

$$K_H = F_1 \cdot z, \text{ where } F_1 \text{ is a constant of proportionality.} \quad (\text{III-32})$$

Then

$$\frac{z_1}{K_{Ht}} = \frac{\delta_H}{K_\delta} + \frac{z_1 - \delta_H}{F_1 \cdot \frac{z_1}{2}}$$

or

$$K_{Ht} = \frac{z_1 K_\delta F_1 \frac{z_1}{2}}{F_1 \cdot \frac{z_1}{2} \cdot \delta_H + (z_1 - \delta_H) K_\delta} \quad (\text{III-33})$$

Now consider Figure III-14b. In this diagram, the same heat flux as before is taking place across the zone z_2 to z_3 which is wholly in the turbulent region, and therefore it may be written that

$$q_c = \rho c_p \frac{K_{H(2-3)}}{(z_3 - z_2)} (T_2 - T_3) \quad (\text{III-34})$$

Since the heat flux is the same in both a and b,

$$\frac{K_{Ht}}{z_1} (T_s - T_1) = \frac{K_{H(2-3)}}{z_3 - z_2} (T_2 - T_3)$$

Therefore

$$K_{Ht} = K_{H(2-3)} \frac{(T_2 - T_3) z_1}{(T_s - T_1) (z_3 - z_2)} \quad (\text{III-35})$$

where $K_{H(2-3)} = F_1 \cdot \frac{(z_2 + z_3)}{2}$ from equation III-32.

The two unknowns in general are K_{Ht} , which is given by III-35, and δ_H which may be given from III-33, after K_{Ht} is found. Alternatively equations III-33 and III-35 may be combined to give

$$\delta_H = \frac{z_1 K_\delta}{\left(F_1 \cdot \frac{z_1}{2} - K_\delta\right)} \cdot \left[\frac{z_1}{z_2 + z_3} \cdot \frac{\frac{(T_s - T_1)}{z_1}}{\frac{(T_2 - T_3)}{(z_3 - z_2)}} - 1 \right] \quad (\text{III-36a})$$

In most cases K_δ will be small in comparison with $F_1 \cdot (z_1/2)$, and can be ignored in the first term on the right-hand side.

$$\delta_H = \frac{2K_\delta}{F_1} \left[\frac{(z_3 - z_2)}{(z_3 + z_2)} \cdot \frac{(T_s - T_1)}{(T_2 - T_3)} - 1 \right] \quad (\text{III-36b})$$

A similar procedure can be carried through using the momentum coefficients, and equating the shear stresses, to give the boundary layer thickness from the point of view of momentum.

In this case

$$\delta_M = \frac{2K_{\delta_M}}{F_2} \left[\frac{(z_3 - z_2)}{(z_3 + z_2)} \cdot \frac{u_1}{(u_3 - u_2)} - 1 \right] \quad (\text{III-37})$$

where $K_{\delta_M} = \nu$. In actual practice, it is easier to measure average velocities than average temperatures when the ground surface temperature is involved. Since there is much more doubt about the variation of K_H with height than there is regarding K_M , it will probably be more realistic to evaluate δ_M than δ_H . It is not proposed to enter into a discussion of the identity of these two quantities here save to note that if they are equal, then the temperature profile can be similar to the velocity profile only if K_H is everywhere equal to K_M , since K_{δ_H} has the same magnitude as ν .

When the boundary layer thickness has been approximately established in this fashion, perhaps by using temperature and wind velocity data at several different levels, it can be compared with the roughness height calculated for the particular situation. If the roughness height is greater than $2/15 \cdot \delta_M^*$, according to notes from hydraulic lectures by A.T. Ippen, M.I.T., then rough surface flow is taking place, and the applicable equations will probably have been used already. If the roughness criterion is not satisfied, however, and if the roughness height moreover is less than $1/120 \cdot \delta_M$, then smooth surface flow must be taking place, and the data should be tested further for this state at conditions of natural stability. It may be possible in this way to determine over what wind velocity and temperature gradient conditions the transition

*It must, of course, be remembered, as is mentioned earlier, that z_0 , the roughness parameter in meteorology, is not equivalent to ϵ , the equivalent sand roughness of Nikuradse's⁷⁴ experiments. It has been established¹¹² that $z_0 = \epsilon/30$ approximately. The original criteria are that the effects of the rough surface begin to be felt when $\epsilon = \delta_M/4$ and increase until rough surface flow is fully developed at $\epsilon = 4 \cdot \delta_M$. Substituting z_0 , the conditions become

smooth $z_0 < \delta_M/120$

fully rough $z_0 > 2/15 \delta_M$

from smooth to rough surface flow takes place. As can be seen from Figure III-9, in such work as this it is of relatively little importance which type of flow is considered in the derivation of the eddy diffusion coefficient when the roughness is very small.

As an example, Table III-8 is given to demonstrate the applicability of equation III-37.

Table III-8
Data from Seabrook, New Jersey, 21 August 1952

z_0 was calculated to be 0.017 cm

Time	Ri	K_{M60}	K_{M80}	u_{160}	$\left(\frac{\Delta u}{\Delta z}\right)_{40-80}$	δ_M (cm)
0730	-22.3	596	794	338	0.525	0.132
1030	-14.2	879	1173	580	0.975	0.081

It will be seen from the use of the smooth-rough criteria that at 0730 the flow is near the end of the transition stage and approaching rough surface flow, and that at 1030 fully rough flow is occurring. As this value of z_0 represents a very low value of roughness indeed for a natural surface, it is unlikely that smooth flow will occur in nature at other places, though it may be possible over a concrete runway, for which profile data is lacking. Probably an airport runway in any other direction but directly downwind is not long enough for fully smooth surface flow to develop in any event. The wind velocities necessary to produce smooth surface flow are generally so small that no practical measurement of them can be made.

However, to return to Figure III-13, it is proposed to discuss briefly the curve obtained from the viewpoint of the subsequent analysis. It will be noted that at moderate stabilities and instabilities, the transfer coefficient K_{Ht} over the whole range of interval 0-160 cm bears an approximately constant ratio to the coefficient existing over the height difference 40-80 cm. (A slightly different constant appears to exist on each side of the line denoting neutral equilibrium.)

This would appear to indicate that K_{Ht} is more dependent on the movements taking place in the turbulent zone than in the laminar sublayer. Reason substantiates this since in this area of the plot both large stabilities and instabilities are usually prevented from developing by moderate to high winds. Over a relatively

large range of velocities they probably cause the laminar sublayer to remain at a fairly constant small value, contributing to a small degree only to the processes of heat transfer. However, in the zone of high stability, large boundary layers develop (since these usually occur at times of strong outgoing radiation and low winds) which have a large influence on the transfer of heat, while the turbulent zone plays a minor part. Again, when highly unstable conditions occur, the flow and heat-transfer processes which are taking place are those tending towards natural convective motion with low wind velocities in a horizontal direction, and the boundary layer again assumes an increasingly important role. At this stage and for this discussion it is not thought desirable to pursue the matter further, but it should prove of interest to calculate the boundary sublayer thickness for each point in Figure III-13 and plot the result versus, say, the Richardson number for the point.

3-17. Programming Procedure Preliminaries. Having evaluated various formulae and having drawn curves from the available data, it remains to point out how they can be used to estimate the heat flow under given conditions, and how to set up the programmer of the computer to take into account the relevant factors. It is recommended that the following procedure be carried out, with modifications depending on the circumstances.

If only a few days data are to be programmed into the computer in order to investigate soil temperatures near the surface, a plot should be made of air temperature at the 160-cm level versus time. To this plot should be added an estimation of stable and unstable conditions, from information on clouds and time of day. On the same time scale another graph of the variations in wind velocity at 160-cm above the surface over the time interval, together with an approximate evaluation of the surface roughness from Table III-5, must be made. Then Figure III-9 can be entered to make another diagram of the variation in K_{60} with time. Thus the momentum eddy diffusion coefficient is found for all times during the period in question.

Next, it is necessary to use Figure III-11 in conjunction with the estimated stabilities to arrive at the thermal eddy diffusion coefficients, which can be plotted on the same scale as the momentum coefficient. The latter is adjusted in magnitude over the unstable periods only. Thus the stage has been reached where the variation in K_{H60} is known over the interval considered, and it only remains to find the value of eddy diffusion coefficient of heat over the whole range from ground surface to 160-cm height. This is accomplished by means of Figure III-13 from which it is noted approximately that, in moderately unstable conditions,

$$\frac{K_H (0-160)}{K_H (40/80)} = 0.22 \quad (\text{III-38a})$$

and, in moderately stable conditions,

$$\frac{K_H (0-169)}{K_H (40-80)} = 0.38 \quad (\text{III-38b})$$

It is considered unwise to place too great reliance on the data obtained at extreme values of the Richardson number or to attempt a more precise identification of a curve through the plotted points. The abrupt change at neutral equilibrium is to be regretted, but the reader is referred to the different expressions holding for the heat flow to or from flat plates in laboratory conditions depending on whether the plate is at a temperature above or below that of the surrounding atmosphere.

When the value of K_{HO-160} has been plotted on the time scale adopted, it needs only to be multiplied by the factor $\rho c_p/160$, which is essentially constant, to render it into a form suitable for transformation into hydraulic conductance. For short periods it will generally be necessary to vary continuously the value of conductance for the correct operation of the analog.

The above procedure will be carried through in Part V for a case in nature for which reasonably reliable heat-flow data already exist. Since it is inconvenient to adjust the hydraulic resistances continually, some averaging techniques will also be dealt with in that part.

PART IV - RADIATION EFFECTS

4-01. General. In Part II, when the various sources of heat flux at the air-earth interface were discussed briefly, some mention was made of the part which radiation of various types played in the heat balance. It is now proposed to discuss this aspect of the problem more thoroughly, so that a clear understanding of radiation processes will be obtained, and so that it will be possible to compute the heat flux due to radiation at a given locality.

Again the U.S. Weather Bureau information will be relied upon as the principal source of data on the meteorological conditions prevailing at a location over a period of time. For the necessary computations, the latitude, longitude, elevation, and soil properties of the site will be given, and the remaining information must be obtained from weather summaries. As before, the problem may present itself in two ways: either as a prediction of surface and subsurface temperatures over relatively short periods of a few days, or over periods months in extent. In the first case, quite accurate values of the necessary variables are required, and in the second, average or extreme values over a number of years will be sufficient to give the mean conditions prevailing at a locality, or to give the worst design conditions over a period. The worst conditions may refer to deepest thaw or deepest freeze, depending on the area involved and the type of construction to be considered. Because of the nature of the surface temperatures of the two bodies involved, the sun and the earth, the phenomena of radiation submit themselves readily to division and consideration in two parts: shortwave radiation and longwave radiation.

Each type of radiation will be considered in turn, and the amount of information necessary to calculate the energy resulting at the earth's surface therefrom will be investigated and noted. Where possible, curves and charts will be given for the determination of appropriate quantities.

4-02. Solar Radiation. The energy in the earth's atmosphere comes from the sun, and the energy arrives at the outer envelope of the earth in the form of radiation of all wavelengths in the spectrum. Planck's radiation formula shows that the amount of radiation at a given wavelength depends on the temperature of the emitting black body, and Wien's displacement law demonstrates from Planck's formula that there is a wavelength for which the radiation intensity is a maximum for any given temperature. This means, in effect, that the higher the temperature of the body emitting radiation, the shorter is the wavelength at which the maximum radiation intensity occurs.

In the case of the sun, which may be regarded as a black body with a surface radiation temperature of about 6000°K ,* the maximum intensity of radiation occurs at a wavelength of about 1.5 microns. This is between green and blue in the visible part of the spectrum. Almost the whole energy of the sun's radiation is transferred in the range 0.15 to 4μ (microns).

However, the earth, with an average surface temperature of about $287\text{--}300^{\circ}\text{K}$, emits its maximum intensity of radiation at about 10μ wavelength. An arbitrary division is made at a wavelength of 3μ between shortwave radiation, $0\text{--}3\mu$, on the one hand, and longwave radiation, 3μ to effectively 80μ , on the other. Consequently, as the sun and earth, looked at as emitting bodies, give out radiation at wavelengths which practically do not overlap, the two types of radiation may be considered separately as shortwave and longwave radiation.

It is of course well known that the shorter the wavelength of radiation, the greater is the penetrating power of the rays (as for example X-rays, which have a wavelength of about $1\text{--}2 \times 10^{-4}\mu$ and ultraviolet rays used in special types of film for taking astronomical photographs). It follows, therefore, that the two types of radiation will exhibit different properties of penetration, or absorption, in various media. These properties can be dealt with for each radiation band by itself.

A great deal of work has been done on the evaluation and correlation of computed amounts of radiation with measured values, and for the purpose of this study it is sufficient to collect and utilize such of the previous formulae and charts as will be relevant to the accuracy of the present work. It is considered necessary, however, to give a brief outline of the concepts which lead up to the theoretical evaluations, in order better to indicate their shortcomings and so that they may be used with a full knowledge of the premises on which they are based.

4-03. Shortwave Radiation. The method of approach to the problem of computing radiation quantities at the earth's surface will be similar to the path taken by the rays themselves: First of all the radiation on a plane normal to the sun's rays at the limit of the earth's atmosphere will be given from current best estimates and then on a horizontal surface at that place; next, the various amounts of attenuation which the rays undergo in reaching the earth's surface will be considered, and expressions and charts for calculating the amount of radiation on a horizontal plane on the surface on

* $^{\circ}\text{K}$. is degrees centigrade on the absolute scale, that is degrees Kelvin $273^{\circ} + ^{\circ}\text{C}$.

a "clear" day at any instant will be given. Finally the depletion due to different amounts and types of cloud cover will be studied.

On a flat plate held normal to the direction of the sun's rays at the mean distance of the earth, outside the atmosphere, the intensity of radiation due to the sun is considered to be $1.94 \text{ cal/cm}^2/\text{min}$, or langleys/min, in the unit which is frequently used in meteorological work. This value is given the symbol I_0 , and is known as the solar constant. It is based on extrapolations of observations made by the Smithsonian Institute in the atmosphere. Recent researches⁵⁶ seem to indicate that a more correct value may be $2.00 \text{ langleys/min}$, which has been used in the past in calculations by Milankovitch.⁷⁰ The value will of course vary as the square of the distance of the earth from the sun which does not remain constant, as the earth is closest to the sun at the winter solstice in the northern hemisphere, and farthest at the summer solstice. The maximum variation from the solar constant mean due to the changing distance is about $\pm 3\%$ which will generally be ignored in computations. However, should it be desirable to take this into account, the meteorological tables issued by the Smithsonian Institute in Washington give values of the ratio of the solar constant holding at any time to the mean value.

When the sun's radiation passes through the atmosphere, certain wavelengths, or wavelength bands, are selectively absorbed by constituents in the atmosphere. The sun's spectrum viewed from the earth's surface demonstrates these bands by black lines called absorption lines in the visible portion, indicating an absence of radiation of this particular wavelength. The principal absorbing agent is water vapor, and thus the amount of radiation reaching the earth's surface will depend on the amount of water vapor in the path which the ray has to traverse. A ray passing through the earth's atmosphere may suffer depletion either by this absorption, in which case the ray's energy is transferred to the absorbing medium and lost to the earth's surface, or it may be scattered by particles of matter in the air, in which case some of the radiation will still reach the ground and impart its energy to it. However, it will arrive in the form of diffuse radiation; thus both direct and diffuse solar radiation must be taken into account in energy evaluations.

4-04. Absorption. If the radiation is passing through an absorbing medium in which depletion takes place according to the exponential law, and the intensity after passing through a layer of unit thickness normal to the ray is

$$I = I_0 p \quad (\text{IV-1})$$

then a ray passing through the unit thickness at an angle Z (zenith angle) to the normal will have an intensity

$$I = I_0 p^{\sec Z} \quad (\text{IV-2a})$$

Because of refraction and the curvature of the earth the path of the ray will be slightly curved, to an extent depending on the original zenith angle, and equation IV-2a can be rewritten

$$I = I_0 p^m \quad (\text{IV-2b})$$

Then, if the length of path of a ray measured along a line perpendicular to the earth's surface at sea level is taken as unity, m represents an 'air mass' or a ratio of the actual path length to a reference length considered as the thickness of the atmosphere vertically above sea level. The Smithsonian Meteorological Tables¹⁰⁰ give values of m for various angles of Z , which correspond closely to $\sec Z$, except for Z angles above 70 to 80°.

Of primary interest to the present work is that proportion of the radiation which falls on a horizontal surface, i.e. the vertical component, and this is given by the equation

$$I = I_0 p^m \cos Z \quad (\text{IV-3})$$

Naturally the zenith angle of the sun varies during the day, and an expression to find the zenith angle is given by spherical trigonometry

$$\cos Z = \sin D \sin L + \cos D \cos L \cos t \quad (\text{IV-4})$$

where Z is the zenith distance (angle),

D is the sun's declination,

L is the latitude of the place of observation, and

t is the hour angle of the sun.

Then, in the course of a day, $\cos Z$ and m will vary continuously as the sun moves across the sky. To obtain the total radiation during a day it is necessary to consider equation IV-3 as holding at one instant, when $I = dq_{ro}/dt$. $\cos Z$ is given by

equation IV-4, in which both D and t are time functions and equation IV-3 is integrated from sunrise to sunset, or, in symbols,

$$q_{ro} = \frac{1}{t} \int_{t_1}^{t_2} I_0 p^m \cos Z dt \quad (IV-5)$$

remembering that m is also a function of Z . However, equation IV-3 applies strictly to monochromatic radiation and it is here assumed to hold over the entire spectrum of solar radiation.

It must be noted that p^m is a function of the wavelength of the radiation, so that the amount of absorption of a particular wavelength depends on the angle of elevation of the sun, and therefore equation IV-5 cannot be applied with strict accuracy.

When there is no depletion or absorption of the sun's radiation, that is, at the outer limits of the atmosphere, it is possible to calculate fairly simply the amount of solar radiation on a horizontal plane for a particular latitude and time of year. This has been done, and a chart of total daily solar radiation at the top of the atmosphere on a horizontal surface versus time of year and latitude is given in the Smithsonian Meteorological Tables¹⁰⁰ (hereafter referred to as S.M.T.). Since p is a transmission coefficient always less than unity, it is also possible, with no knowledge of the mechanism by which the depletion occurs, to evaluate equation IV-5 for values of p equal to 0.6, 0.7, 0.8, 0.9, and 1.0. Originally this was calculated by Milankovitch, and the data are presented in tabular form in the S.M.T. as daily totals of direct solar radiation reaching a horizontal surface on the earth versus latitude and time of year. It is necessary to make the qualification "direct solar radiation" since, once the rays have passed through an atmosphere, they may be scattered as well as absorbed. Some of the scatter is reflected to the surface of the earth as diffuse solar radiation which must be reckoned with separately.

As values of solar radiation can now be obtained for different values of p , it remains to consider how an appropriate transmission coefficient may be arrived at. It was mentioned previously that the processes of absorption in the earth's atmosphere were principally a function of water and dust content, with some absorption due to ozone and carbon dioxide. A great deal of work was done by Fowle³⁹ and Kimball⁶¹ (1931) on the relations between the quantities of agents present in an atmosphere, and the resulting depletion of radiation. Moon,⁷² using these results, considered atmospheres containing varying amounts of water vapor and dust and gave a method of calculation of solar irradiation curves for any elevation of the sun. By using this method and by

adopting a standard atmosphere, Moon gave spectral irradiation curves at sea level, which are utilized by the American Society of Heating and Air Conditioning Engineers (ASHAE) in their Guide.⁶

Applying Moon's calculations to winter conditions, Hutchinson and Chapman computed curves showing direct normal incidence solar radiation as a function of the solar altitude. Threlkeld and Jordan¹¹⁰ present some composite curves taken from both the other investigators, together with comparisons with measured irradiation at local sites. These curves are long-term average curves, and cannot be used for actual local determinations with given meteorological conditions.

For computations in this latter category, one of two methods can be chosen. Either the data for solar irradiation on a horizontal surface at ground level for different values of p taken from S.M.T. can be used, together with a determination of the average transmission coefficient prevailing over a period; or evaluations of equation IV-5 can be carried out for a particular point, using recorded values of the water vapor content of the air. In this case the data will probably have to be plotted, since the determination of the amounts of solar radiation must be carried out graphically, as generally no smooth functions will exist.

In both these methods, the unknown quantity is still the transmission coefficient, and it is necessary to determine in what way it depends on the properties of the air. In the work of Fowle and Kimball referred to above, the usual way of presenting the data was to refer the transmission coefficient to the amount of precipitable water in the air where the precipitable water is given by the expression

$$w = \frac{1}{g} \int_{z_1}^{z_2} q \, dp \quad (\text{IV-6})$$

taken between the heights z_1 (usually ground level at the point considered) and z_2 , the effective height of the atmosphere; p is the atmospheric pressure. Obviously, this represents an evaluation of the amount of water which would accumulate if all the water vapor in a column of air were precipitated. The evaluation of such an equation is bound up with continuous vertical measurements of specific humidity and atmospheric pressure in the column of air above a point, and is generally impossible.

However, some investigators have examined the relation between such amounts of precipitable water and the humidity at the earth's surface, the most commonly measured vapor variable. Gabites⁴¹ has analysed the work of the various investigators

and proposes the equation

$$w_t = 2.3 e_0 \quad (\text{IV-7})$$

for the depth of precipitable water based on the sea level vapor pressure, where the latter quantity is obtained by an expression

$$e_0 = e_z (1 + 0.0004 z) \quad (\text{IV-8})$$

This gives, then, the amount of precipitable water in the air column above sea level. It is necessary, for solar radiation quantities, to calculate the amount of precipitable water in the air above the elevation of the ground surface in question only.

If the total amount of precipitable water in a column of air 0-8 km high is considered to include most of the water in the entire depth of the atmosphere, it is desired to estimate the amounts existing above various levels contributing to the total. According to Gabites, Shands gives the following quantities:

$$\begin{array}{ll} \text{above 1 km} & w = 0.65w_t \\ \text{above 2 km} & w = 0.40w_t \\ \text{above 3 km} & w = 0.25w_t \end{array} \quad (\text{IV-9})$$

which give satisfactory agreement (using radiosonde data) for different stations and times of year. Equations IV-7, IV-8, and IV-9 can be combined to form a convenient chart, Figure IV-1, for the estimation of precipitable water above a point, given the elevation of the point above sea level and the humidity at the point in mm mercury vapor pressure.

It has been found that the amount of precipitable water in the atmosphere affects the transmission coefficient to an extent dependent also upon the atmospheric pressure, but Gabites points out that this effect is not large and may be neglected.

To return to the original intentions of this section, if the vapor pressure at ground level (or fairly close) is known at a particular time, it is desired to calculate the transmission coefficient for the air mass existing at that time. Again, the S.M.T. supply useful curves for the determination of this value, in the form of a chart of the transmission coefficient for radiation through moist air versus the air mass and the amount of precipitable water present in the air. It now appears for the determination of equation IV-5, that, assuming a constant value of the amount of

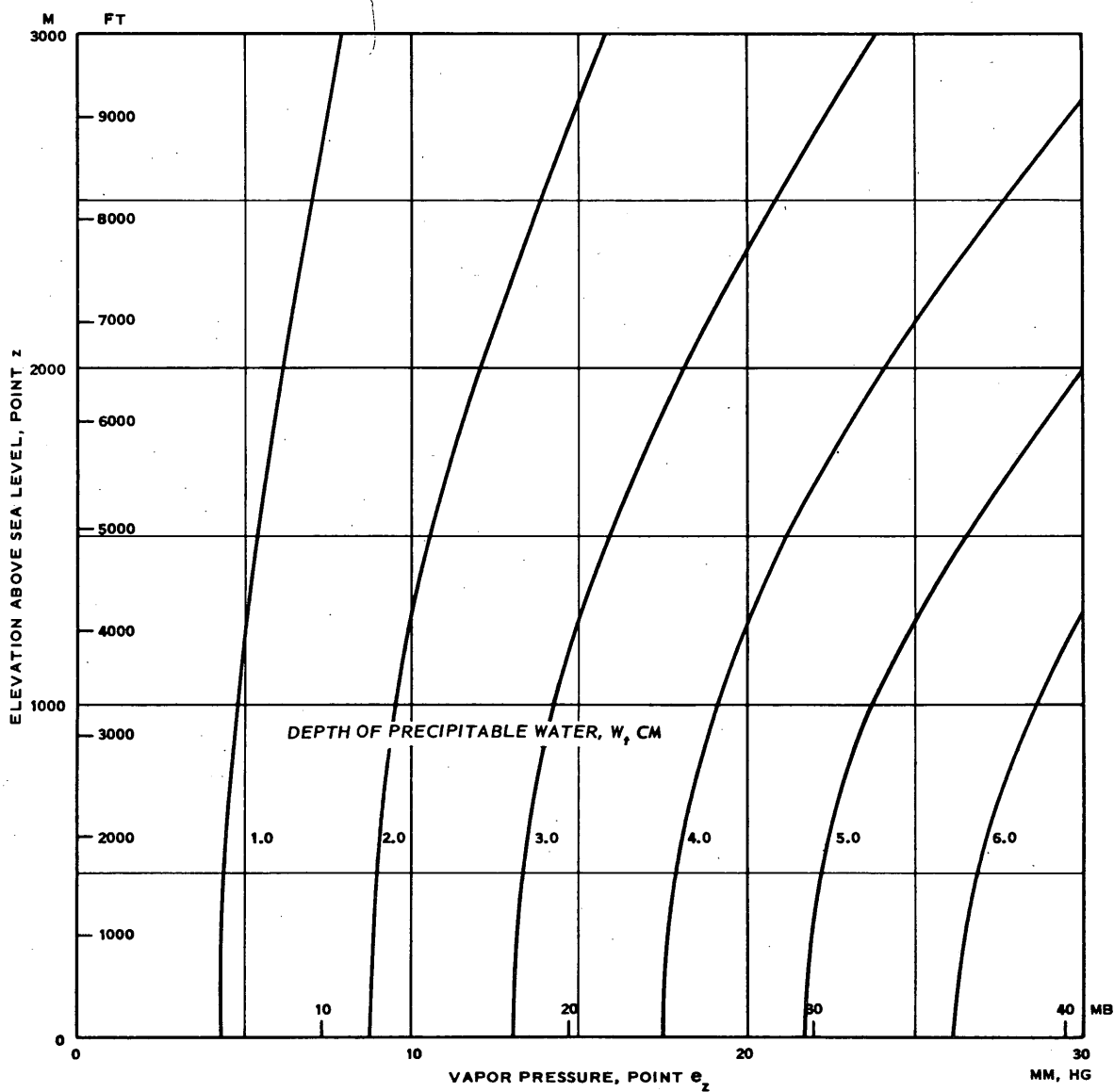


Figure IV-1. Humidity chart.

precipitable water vapor present in the atmosphere, p depends on the air mass also, so that in the evaluation of the integral both p and m vary with Z , which renders the computation laborious indeed.

It may be simplified, however, by taking an average value of $\cos Z$ for use in the calculations. This average value will yield a value for m ($= \secant Z$) and thus for p in turn. Since the maximum irradiation intensity occurs at and around noontime and since m is large for a relatively long time in the morning and at night, it would not be correct merely to calculate an average for m , as is pointed out by Gabites. Then the values of cosine Z , m , and p can be used for the easy analysis of equation IV-5. Gabites has calculated values, $\bar{m}$ of a mean for m , and gives monthly mean values of $\bar{m}$ versus latitude in a table. He also shows, by graphical integration of equation IV-5, when all the quantities vary, that the calculation of daily insolation by the mean method gives results in substantial agreement with the correct (graphical) values.

4-05. Diffuse Radiation. So far the calculations have given the amount of direct solar irradiation on a clear day in a known location, and it remains to calculate the amount of radiation which arrives at the earth's surface in diffuse form. As has been said before, the sun's rays undergo both scattering and absorption. The absorption has been dealt with, and the scattering is caused by the impingement of the rays on dry air molecules, water vapor, and dust particles. If the particles which cause the scattering are small in relation to the wavelength of the radiant beam, a postulation of Rayleigh's indicates that one-half of the total amount of radiation affected by scattering will be reflected back to space or absorbed in the atmosphere over repeated reflections, and one-half will reach the surface of the earth as diffuse radiation. It thus remains to find out what proportion of the radiation is affected by scattering.

In the S.M.T. in the table dealing with the actual direct radiation reaching the surface of the earth, the transmission coefficients include both scattering and absorption, and in the table giving the transmission coefficient under appropriate water vapor conditions, the amount of depletion considered includes scattering also. If the extra-terrestrial radiation on a horizontal surface is found for the time and place considered and is decreased to allow for water vapor and ozone absorption, the difference between the resulting total and that found from the S.M.T. for the radiation arriving on a horizontal surface at ground level must be due to scattering. Thus, in

the S.M.T. a suggestion by Fritz is noted that, in order to obtain the quantity of diffuse radiation on a horizontal surface on a clear day, the following steps should be carried out:

- (1) Find the extra-terrestrial radiation for the latitude and date required.
- (2) Decrease (1) by 7 percent for water vapor absorption and 2 percent for ozone absorption to get 0.91 times (1).
- (3) Find the appropriate value of direct radiation reaching the surface of the earth from the S.M.T., and subtract it from 0.91 times (1). The resulting difference, S , approximates the energy scattered out of the solar beam.
- (4) Find $S/2$. Then $S/2$ must be added to the amount of direct radiation reaching the surface of the earth, found in (3) to give the total irradiation.

A further method of calculating the amount of diffuse radiation falling on a horizontal surface has been presented recently by Parmelee.⁷⁶ This method is based principally on the results of radiation measurements at Cleveland, Ohio, but appears to hold promise of general applicability. In a chart, Parmelee presents values of diffuse radiation on a horizontal surface versus direct radiation on a horizontal surface, solar altitude, and a clearness ratio. The clearness ratio is an arbitrary parameter calculated in the following way: As mentioned above, Moon⁷² gives curves of solar radiation on a clear day for a standard atmosphere. For any given location and time, these curves will give a value of irradiation if, at the same elevation, the measured solar radiation (or that calculated on the basis of known atmospheric properties) is known, then the clearness ratio is the proportion of measured radiation to Moon's value, which gives an idea of the clarity of the atmosphere.

Parmelee's values have been approximated and tabuled in the ASHAE Guide⁶ for different altitudes of the sun and for two extreme cases: that of a clear atmosphere, as in the central midwest states, and that of an industrial atmosphere. These values can be used together with the direct solar radiation on a horizontal surface given by the tables (by calculation from that for normal incidence) to find the total radiation on a horizontal surface on a clear day.

Two more possibilities remain to be considered: partially cloudy and completely overcast days. In these cases it is still usually necessary to calculate the amount

of total radiation on a horizontal surface assuming a clear sky, and apply a ratio of cloudy sky to clear sky radiation in the computations.

4-06. Partly Cloudy Skies. It has been generally postulated by observers that the relation between total cloudy sky radiation and total clear sky radiation on a horizontal surface is of the form

$$\frac{q_{rc}}{q_{rs}} = a + b S \quad (IV-10)$$

in which different observers have obtained different values of the constants a and b . This equation depends essentially on whether the data considered are daily radiation quantities or monthly averages.

If the daily quantities are used in the computations, a plus b must equal one, since the possibility of a completely clear day with S equal to unity exists. On the other hand, using monthly means, there is very little likelihood of obtaining one clear month, and the sum of a and b need not be unity.

Where data involve not the percentages of possible sunshine, but the cloud cover in tenths or a percentage, a relation such as that given by Gorczynski⁴⁶ must be used.

$$S = (100 - C) (1 + 0.005 C) \quad (IV-11)$$

where both S and C are given in percent.

According to different authorities, the values of the constants in equation IV-10 are

Authority	a	b	
Angstrom ⁹	0.25	0.75	
Kimball ⁶¹ (1927)	0.22	0.78	(IV-12a)
Kimball ⁶¹ (1928) gives $q_{rc}/q_{rs} = 0.29 + 0.71 (1 - C)$ with C in tenths			
Haurwitz ¹⁰⁰	0.22 - 0.30	0.78 - 0.70	

Now Fritz and MacDonald,⁴⁰ and Black, Boynton, and Prescott¹⁶ have studied monthly averages of solar radiation and percentage sunshine as actually observed, and they obtain values of the constants differing from those given above.

Authority	a	b	
Fritz and MacDonald	0.35	0.61	
Black, Boynton, and Prescott	0.23	0.48	(IV-12b)

The above compilations of data have all been made from the point of view of relating the amounts of radiation which were observed to take place. Another observer, Gabites,⁴¹ has chosen to investigate data from the point of view of relating the radiation on a cloudy day to the calculated amount of radiation on a clear day, assuming the monthly average value of precipitable water to hold on that day. On this basis he gives a formula

$$\frac{q_{rc}}{q_{rs} \leftarrow (\text{calculated})} = 0.33 + 0.71 (1 - C) \quad (\text{IV-12c})$$

From the point of view of convenience to the calculator, a more useful way of obtaining the necessary information is by reference to a chart or nomograph. Two groups of investigators have supplied such charts, Gerdel, Diamond, and Walsh,⁴³ and Hamon, Weiss, and Wilson.⁴⁸ The first investigation was carried out from the point of view of predicting the amount of solar radiation available to melt snow, and the second indicates that values of radiation given in the first publication are too low by about 50 cal/cm²/day. However, Gerdel, Diamond, and Walsh's paper gives results for calculations involving high latitudes, whereas, with the second reference, it is only possible to compute radiation for latitudes up to 50° N. It may be in order, however, to compute the difference between the two methods for one set of conditions at some values of the latitudes which overlap, and to extrapolate this difference to correct the G.D. & W. paper for the more northerly latitudes.

Then, depending on the period covered by the calculations, a group of equations from IV-12a, b, or c can be used to give the total solar radiation on a horizontal surface on partly cloudy days.

4-07. Completely Overcast Skies. When the sky is completely overcast, the incoming radiation is all of a diffuse nature, although it is not completely omnidirectional (that is, a certain amount of the radiation comes from the direction of the sun) and a completely different situation from the above arises. It is no longer possible to calculate the radiation, and empirical data must be used. The amount of radiation will depend as before on the amount of precipitable water in the earth's atmosphere and will also depend on the type and height of the clouds. Since clouds

of a particular type tend to develop at characteristic heights, the only available curves show the amount of radiation as a function of cloud type only, and the effect of the amount of precipitable water in the atmosphere is also obscured in the data. Thus the radiation is presented in curves showing it as a function of cloud type and air mass only, i.e., the irradiation, as would be expected, is a function of the sun's altitude.

Haurwitz has given these curves in a paper, and they are also duplicated in the S.M.T. 100

An interesting point arises from a consideration of data analysed by Hand and Wollaston¹¹⁵ at Blue Hill Observatory, Massachusetts. They evaluated the ratio of diffuse radiation to total solar and sky radiation over a period of four years at the observatory to find that the ratio is approximately constant at $38\% \pm 10\%$ over the whole year, including cloud effects. This ratio will probably vary from one location to another, but if such a variation were known, it would enable the amount of diffuse radiation to be approximately estimated over a two-weekly or monthly period.

Since it is unlikely that the types of clouds would be known at any location, it will also probably be necessary to choose an average curve from Haurwitz's chart to represent radiation versus air mass for all days which were completely overcast. As the type of cloud most common in different parts of the country will vary, this average curve should, in general, be different for different sections of land, but a great deal of work would need to be done to evaluate the different tendencies.

Finally, some other approximations may be made, or may have to be made, in order to be able to calculate usable data. In some localities amounts of cloudiness or percentages of possible sunshine will not be known, and in those circumstances it will be necessary to obtain crude estimates of numbers of clear, partly cloudy, and completely cloudy days. Then equation IV-12c can be used to give an intermediate value for the ratio of q_{cloudy} to q_{clear} , for say S equals 50% or, approximately

$$\frac{q_{\text{rc}}}{q_{\text{rs}}} = 0.60$$

for the partly cloudy days. An even greater simplification can be made for grouping all the days as either "clear" or "completely overcast" and making all the

calculations on this basis. The results would be crude, but would give indication, at least, of overall tendencies and effects on temperature regimes.

It may be mentioned here that U.S. Government Weather Bureau Technical Papers¹¹⁵ are available which augment the information given in the monthly summaries. These papers give such data as upper air averages of temperature, pressure, and relative humidity; percentages of sunshine and cloudiness at selected stations; daily totals of solar and sky radiation at selected stations in the U.S., Hawaii, and Alaska.

4-08. Albedo. Since a portion of the solar radiation incident on the earth's surface is reflected back to the sky, it is important to have figures for the reflectivity, or albedo, of different kinds of surfaces. This study is particularly concerned with concrete and bituminous surfaces, but it will occasionally be necessary to compare the effective radiation on these surfaces with that on nearby terrain, so that other values are given for comparison. Values for natural surfaces are taken from the S.M.T., expressed as a ratio of reflected radiation to total incident radiation.

Surface	Albedo	
Grass	0.14-0.37	
Bare ground	0.07-0.20	
Black mold	0.08-0.14	
Dry sand	0.18	
Wet sand	0.09	
Snow, ice	0.46-0.86	
Concrete, light color	0.50 ± 0.10*	From Brooks ¹⁸
Bituminous concrete	0.10 + 0.10	
	- 0.05	

The amount of radiation received by the surface as a heat flux can be calculated from the total incident radiation times (1.00 - albedo).

It is noted that it is only possible to give rough values of the albedo in most cases because of the natural variability of the surface, and because it is difficult to classify materials adequately. However, it should be possible to obtain more accurate values for the two types of concrete which are of principal interest in this study. Furthermore, the reflectivity of a surface for a particular radiation also

*However, Wilkes¹¹⁹ gives a value here for reflectivity for solar radiation of 0.17.

depends on the angle of incidence of the ray striking it. It may not be possible to allow for this, because of the large uncertainties involved. For instance, in the UW report on the Great Plains Turbulence Project at O'Neill, Nebraska, values are given of the reflectivity of a short grass surface for shortwave solar radiation in which the reflectivity varies from values of 0.7-0.8 at sunrise and at sunset to fairly consistent values of 0.22-0.23 for the two hours on each side of noon.

It would thus appear that, if very accurate work is to be undertaken, the variation of albedo over the day must be known and taken into account. Since the albedo depends on the direction of the radiation, it follows that it will vary principally for the direct solar rays and will be fairly constant for the sky radiation, although this is most intense near the sun also. On completely overcast days, the albedo will also remain fairly constant. As the penetrating power of radiation of different wavelengths varies, the reflectivity is also a function of the wavelength, and for longwave radiation the ground surface acts essentially as a black body.

Now that the methods of calculating the quantity of heat due to solar radiation have been given for clear, partly clear, and cloudy days under different atmospheric conditions, it is necessary to go into a description of the techniques available for estimating the amounts of longwave radiation which take place during the day and at night.

4-09. Longwave Radiation. The longwave radiational process at the earth's surface has two components, the radiation outwards from the earth's surface, which behaves essentially as a black body, and the back radiation from the atmosphere downwards. The phrase 'nocturnal outward radiation' is frequently employed to describe what takes place, but this is really a misnomer, as longwave outward and back radiation also take place during daylight hours. In daytime it is dominated by the magnitude of solar irradiation and must usually be measured indirectly, whereas the nocturnal radiation can fairly readily be measured directly.

Since the earth radiates as a black body at the temperature of its surface, it is possible to calculate the outgoing black body radiation of the earth's surface quite simply as

$$q_{e} = \sigma T_s^4 \quad \text{(IV-13)}$$

from the Stefan-Boltzmann law. Geiger⁴² has calculated the amount of radiation of the ground surface at various temperatures, and gives the following values:

<u>Ground Surface Temperature, °C</u>	<u>Outgoing Radiation cal/cm²/min</u>
-40	0.244
-30	0.288
-20	0.339
-10	0.395
0	0.459
10	0.530
20	0.609
30	0.696
40	0.792
50	0.899
60	1.015
70	1.143

Geiger points out the danger in assuming, because of its more obvious nature, that nighttime outgoing radiation is greater than that occurring during the day, or that larger amounts of outgoing radiation take place in the Arctic in the long winter night. Only the duration of radiation is greater in the latter case, the unit time radiation is actually smaller. As the amount of radiation outward from the earth's surface can be calculated from the known temperature of the earth's surface, it follows that the net outgoing radiation depends on a determination of the amount of back, or atmospheric downward radiation taking place.

4-10. Clear Skies. Initially limiting the discussion to the case of clear skies, the back radiation will be due to the emissivity of the gases and vapors in the earth's atmosphere. It has been mentioned before that these gases are selectively absorptive; this holds true for longwave radiation as well as shortwave. Consequently, the amount of back radiation reaching the earth's surface from a layer of gas at some temperature is described by Kirchhoff's law and will depend on the wavelength considered. The two gases of most importance in the back radiation process are water vapor and carbon dioxide, the latter contributing about 18% of black body radiation for the temperatures involved.

Because of the complexity of the emission of radiation from these gases, which compels analysis to take account of the radiation in separate wavelength bands, the exact calculation of atmospheric back radiation is a difficult process. The amount

of radiation depends on the quantities of water vapor and carbon dioxide present (although in the latter case carbon dioxide absorbs radiation so completely at one wavelength and so little at others that the layer present need be very thin). For accurate determinations of radiation it is necessary to obtain vapor pressure and temperature profiles in the atmosphere above the surface.

Simpson⁹⁹ made early analyses of the radiation by approximating the absorption bands and "windows"; his work was later amended by Elsasser³² after further experimental studies revealed inaccuracies in Simpson's assumption. Elsasser prepared a radiation flux diagram with which the amount of atmospheric radiation can be computed in given circumstances. Subsequently Wexler¹¹⁸ made observations of nocturnal radiation at Fairbanks, Alaska, and Fargo, North Dakota, noting differences between the observed values and values calculated from Elsasser's chart. A modification³³ of Elsasser's chart subsequent to Wexler's observations corrects most of the discrepancies. Robinson⁹¹ has compared estimates of radiation from the chart with observed radiation quantities at Kew Observatory and concludes that for certain low intensities of radiation the chart overestimates the amount by up to 15%. As a result of his investigations, he has constructed another radiation chart, which is a modified and extended version of the original Elsasser flux diagram. It is said to give more accurate results.

It has been pointed out by Moller* that these charts may be used when a very complete knowledge of the conditions prevailing in and above a particular locality exists, but this will generally be a very rare occurrence. More frequently only values of temperature and humidity at some small distance above the ground surface are obtainable. Some effort has been concentrated in past researches to relate the amount of atmospheric radiation to these easily observed variables. The results of such investigations must obviously be based on statistical analyses and will therefore only give values of radiation substantially correct over extended periods of time of days, weeks, or months duration. They can be used for long-term average computations and can provide useful information in a study such as this.

*Moller says⁶⁹ "any comparison between calculated and observed (net outgoing) radiation becomes impossible if the vertical temperature distribution, especially that part close to the ground, is not known," since a temperature inversion near the ground can reduce the effective terrestrial radiation (net outgoing) to $\frac{4}{5}$ of its normal value, and, under extreme conditions found by Mosby during the polar night, to as low as $\frac{1}{2}$.

The types of equations used in these investigations fall into three groups, each expressing the amount of atmospheric back radiation as a function of temperature and vapor pressure near the ground, say between one and two meters above ground surface. The equations are of the forms

<u>Authority</u>	<u>Equation</u>	
Angstrom ⁸	$\frac{q_{l_a}}{\sigma T_a^4} = a - b e^{-\gamma e}$	(IV-14a)
Brunt ²⁰	$= c + d \sqrt{e}$	(IV-14b)
Lake Hefner ¹¹²	$= f + g e$	(IV-14c)

Values of the constants for the equations are given as follows:

<u>Constant</u>	<u>Range (Various Investigators)</u>
a	0.80 - 1.11
b	0.24 - 0.41
γ	0.022 - 0.163
c	0.43 - 0.68
d	0.029 - 0.082
f	0.740 } Lake Hefner
g	

In addition, Geiger⁴² gives a formula due to Raman, set out similarly to Angstrom's in which

$$\frac{q_{l_a}}{\sigma T_a^4} = 0.77 - 0.28 \times 10^{-0.074 e} \quad (\text{IV-14d})$$

In equations IV-14a through IV-17 the term e expresses vapor pressure in mm of mercury.

Probably the differences in the various constants are due to the different atmospheric compositions from one place to another which cannot be expressed explicitly. The differences between the equations based on the constants given are not large and are of the same order as the error involved in any case.

Then the net outgoing radiation will be expressed, using equations IV-13 and IV-14d as

$$q_{\ell o} = \sigma T_s^4 - \sigma T_a^4 (0.77 - 0.28 \cdot 10^{-0.074e}) \quad (\text{IV-15a})$$

Thus the net outgoing radiation depends on the surface temperature and the air temperature, besides the other factors. Most investigators seem to ignore this difference, and none refer to it in detail. It is quite possible for surface temperature to be 10° C. higher than air temperatures during the day and 5° C. lower during the night. According to the previously given table from Geiger, this can cause a variation in outgoing radiation of about 14% on the side of being too small during the day and about 7-8% too large at night, from the case where T_s and T_a are assumed to be identical. If this latter assumption were to hold, then equation IV-15a reduces to

$$q_{\ell o} = \sigma T_a^4 (0.23 + 0.28 \cdot 10^{-0.074e}) \quad (\text{IV-15b})$$

using the air temperature T_a as the more easily measured quantity. This equation has been plotted by Geiger in a very convenient diagram, the values of which, as Geiger points out, are a little too high. The usual magnitude of $q_{\ell o}$ is about 0.100 to 0.200 cal/cm²/min, and, as indicated by Geiger, it falls about 7-8% during the night, to increase slowly from sunrise to about sunset, when it decreases rapidly to reach an average nocturnal level early in the evening.

Consequently, using air temperatures and humidity measured at the usual meteorological levels, it is possible to calculate the downward atmospheric radiation. Then the soil temperature, however determined, will give rise to calculations yielding the outgoing radiation, and the net outgoing radiation can be found by subtraction.

4-11. Partly Cloudy Skies. Because of the difficulties involved in measuring longwave atmospheric radiation during the day, most studies have been confined to nighttime experiments, and these render it difficult to obtain estimates of cloud heights and amounts. However, Angstrom¹⁰ has put forward the equation

$$\frac{q_{\ell c}}{q_{\ell o}} = 1.0 - aC \quad (\text{IV-16})$$

which relates the amount of net outgoing radiation occurring when the sky is cloudy, covered by C tenths of cloud, to that occurring under the same conditions of temperature and vapor pressure if there were no clouds present. The equation distinguishes between high clouds, which have little or no effect on outgoing radiation, and low

clouds, which have a considerable effect, by means of the factor a . Philipps⁸¹ gives values of a depending on the height of the cloud as follows:

Base of clouds, km	1.5	2	3	5	8
a	0.087	0.083	0.074	0.062	0.045

Then the effective outgoing radiation under various cloud heights and amounts can be found fairly readily. Another equation was put forward by Anderson, from data obtained at the Lake Hefner area. He suggests

$$\frac{q_{\ell a}}{\sigma T_a^4} = a + be \quad (\text{IV-17})$$

where

$$\begin{aligned} a &= 0.740 + 0.025 C_e^{-0.0584z} \\ b &= 0.00490 - 0.00054 C_e^{-0.060z} \\ \text{and } 1600 \leq z \leq \infty \text{ in feet} \end{aligned}$$

but these constants must probably be considered to hold for the one area only, unless other localities with similar cloud types and air masses are used for analyses. This equation, although difficult to handle, is also valid for the cases when the sky is completely overcast, when it will, in all probability, be more generally applicable.

4-12. Completely Overcast Skies. Making C equal to 10 in equation IV-16 (the sky is completely overcast) results in an amount of net radiation, when varying from 13 to 55% of the clear sky radiation for different cloud heights.

The same procedure with equation IV-17 gives the following values of the constants:

$$\begin{aligned} a &= 0.740 + 0.25e^{-0.0584z} \\ b &= 0.0049 - 0.0054e^{-0.060z} \end{aligned}$$

Then the equation could be plotted with $q_{\ell a}/\sigma T_a^4$ as a function of vapor pressure and cloud base height.

According to the Lake Hefner report, the equations of atmospheric radiation in either clear sky or cloudy sky conditions give acceptable results as long-term averages, but actual measurements of radiation are for accurate determination of quantities such as evaporation, or local heat budgets, over a period of hours or one or

two days. One further interesting piece of information is given by the report concerning the variation in daily and nightly amounts of atmospheric radiation, which was measured indirectly by day and directly by night. It was found that the atmospheric radiation remained approximately constant at night, diminished during the day to a minimum at solar noon, and then rose again to sunset. As it is expected that the amount of radiation would be roughly constant, increasing slightly during the day as the lower air layers warm up, it was assumed that errors in the instrumentation existed, a fact which was not proved conclusively, and the value of atmospheric radiation was assumed to remain constant at the nighttime value throughout the twenty-four hour period.

4-13. Summary. This part of the report has presented methods for determining the amount of solar radiation reaching and affecting the temperature regime of the ground surface under most atmospheric conditions at any location and time. The values and equations on which most emphasis has been placed are those leading to estimates of fairly long-term averages (weeks to months) of radiational quantities. The data are all easily accessible. Consideration has also been given to the quantities of heat loss to the ground surface by longwave radiational exchange processes with the atmosphere under various air conditions, and equations have been given for the computation of the relevant radiational heat fluxes.

During the course of these studies it was learned that Mr. M.H. Halstead* had been working on a computing device for radiation evaluation. Evidently the apparatus requires manual setting to the latitude, date, hour, surface cover, temperature, and atmospheric moisture or cloud cover, after which it will simulate records of insolation, back radiation, and net radiation exchange. According to the author it gives results which agree reasonably well with radiometer records. Presumably it solves equations of the types given in this part (IV) with appropriate assumptions regarding the constants. The device was built as the first stage of a larger computer which would be used for the prediction of other micrometeorological variables.

*M.H. Halstead, "A meteorological simulator for radiation," Bull. Am. Met. Soc., V. 35, No. 444, November 1954. Abstract of paper given at 134th National Meeting, December 28-30, 1954.

PART V - UTILIZATION OF WEATHER DATA

5-01. General. Now that methods have been given for arriving at air-ground heat-transfer coefficients, solar and longwave radiational heat fluxes at the ground surface, and since annual temperature cycles and wind velocities can be obtained from meteorological summaries, the question of applying the information to the manipulation of the analog computer arises. In the hydraulic analog computer, temperature is represented by height of water (or water pressure), heat flow is represented by water flow, and heat conductance is represented by water conductance. It follows that the previous quantities: air temperature, air-ground heat-transfer coefficient, and radiational heat fluxes, can all be programmed into the apparatus by a suitable raising and lowering of an "air temperature" reservoir, adjusting a hydraulic resistance, and injection of appropriate quantities of water into the part of the apparatus representing the ground surface.

But these may not all be **the most convenient** ways of carrying out the programming and it is necessary to **consider** modifications. That is to say, it is possible to raise and lower a reservoir automatically to follow the curve of air temperature with time, but it is not so desirable to continually adjust the value of hydraulic resistance to correspond to the continually changing heat-transfer coefficient. In addition it is not altogether convenient to meter a quantity of water into the apparatus to simulate reasonably closely the variation in radiant heating or cooling of the ground surface. It is proposed then to investigate the following possibilities:

a. Calculating an equivalent air temperature from the existing air temperature, surface transfer coefficient, radiation, and albedo information which will give a heat flux through the ground surface due to all of these influences together, and which can then be programmed together with the heat-transfer coefficient;

b. Estimating the ground surface temperature from all the given and calculated data, in order to program it directly into the apparatus with no surface transfer coefficients;

c. Finding the proportional effects of air temperature, surface heat-transfer coefficient, and radiation components on the heat regime of the ground in order to simplify the procedures for more rapid, if less accurate, computations;

d. Using an average value of wind velocity, or of surface heat-transfer coefficient to represent the air-ground conductivity over daily or monthly periods.

5-02. Equivalent Air Temperature. For some time, heating engineers, endeavoring to calculate heating and cooling loads on buildings, have utilized an effective air temperature, called the "sol-air" temperature,⁶⁸ in order to take into account the effect of solar radiation on a building surface. Following their method, an effective air temperature will then become the sum of the existing air temperature plus an additional temperature involving the radiational heat flux. The expression

$$T_E = T_a + \frac{q_r \cdot b}{h} \quad (V-1)$$

gives the effective air temperature as used by the heating engineers. In equation V-1 the amount of radiation reaching the surface in question is modified by the absorption coefficient of the surface (1.00 - albedo) and divided by the heat-transfer coefficient for the air space between the point at which the temperature is measured and the surface. Thus it will be seen in cases typical of the present study that for a surface temperature T_s (say lower than the air temperature), the heat flux from the air to the surface will be given by

$$\begin{aligned} q_c &= h (T_E - T_s) \\ &= h \left(T_a + \frac{q_{r,1}}{h} - T_s \right) \\ &= h (T_a - T_s) + q_{r,1} \end{aligned}$$

In this manner the flux of heat due to radiation is clearly seen.

Now the term for radiation influence must be the net inward radiation to the ground, since it represents a heat gain to the surface. Consequently the amount of shortwave downward radiation must first be estimated, multiplied by the absorptivity of the surface, and added algebraically to the estimated longwave outgoing radiation to give the correct flux quantity.

If this is done in order to program the hydraulic analog computer for short periods of time, it is necessary to combine the information of the varying net inward radiation with time and the value of heat-transfer coefficient. This gives a correcting temperature which will be added (algebraically, since outgoing radiation will predominate at

night) to the known existing air temperature to give the effective air temperature which is programmed together with the air heat-transfer coefficient. Over longer periods of time, an estimation of net weekly or monthly inward radiation can be made and utilized in the same manner.

In both cases, the problem of calculating the longwave outgoing radiation must be met, since the ground temperatures are not known. Over short periods of time it is more necessary to calculate it accurately since its temporal variation may have a marked effect on the solutions, particularly at night. In this case it may be possible to install a valve on the apparatus with an outflow approximately proportional to the fourth power of the temperature of the surface, or an Elsasser chart may be used, if enough information is given to estimate atmospheric radiation. Another method would be to use the approach outlined in paragraph 5-03 to calculate the approximate surface temperature, which would give the outgoing radiation, in conjunction with atmospheric data. This could be used in the analog to give a more correct solution, utilizing a second cycle of operations if the first estimate of longwave radiation proved much in error.

Over periods of a few months, probably an accurate enough measure of the outgoing longwave radiation could be obtained from Brunt's or Angstrom's equations IV-14b or IV-15a, using the air temperature as given in equation IV-15b with suitable modifications for clouds, as described subsequent to the above equations in Part IV. It would also be possible to calculate the approximate average ground surface temperature by a method similar to that given in paragraph 5-03 but used over a longer period of time. Again, a second cycle of computations could be made if the actual computer-produced values of surface temperature proved widely different from those obtained in the more approximate solution.

5-03. Approximate Calculation of Ground Surface Temperature. A great deal of work has been carried out and described in the literature for the purpose of estimating the ground surface temperature under certain given conditions. Most of the effort has been expended on calculations involving only one night or at most one day, since the importance of the problem lies, for most meteorologists, in predicting the onset of ground frosts with the accompanying changes of crop damage. If the equation for heat flux at the earth's surface is written

$$q_{so} = -q_{\ell o} - q_{ro} - q_{co} - q_{eo} \quad (\text{III-1a})$$

or

$$k \left(\frac{\partial T}{\partial z_0} \right) = -q_{l0} - q_{co} - q_{eo}$$

for nighttime conditions, the work of the various investigators can be discussed briefly.

Each one has investigated one or more special cases of the equation above. Brunt¹⁹ assumed q_{l0} constant, q_{co} zero, and q_{eo} zero, with the initial condition of uniform temperature distribution in the ground, and found an expression for the temperature distribution in the ground (and therefore at the surface) as a function of time. Brunt¹⁹ also analysed a whole twenty-four hour cycle of radiational heating and cooling in the ground, to arrive at a nonuniform temperature distribution in the ground at the beginning of the night. Philipps⁸¹ investigated a nighttime case where q_{co} was not equal to zero, but made some questionable assumptions regarding its variation. Recently Groen⁴⁷ discusses the previous solutions and considers more involved, and therefore more realistic, cases in which temperature inversions arise at the ground surface. The difference between his solution and Brunt's is not great for periods of only one night, but for long polar nights a marked discrepancy in surface temperature becomes apparent. Groen refers to cases of nonuniform initial temperature distribution, conductivity of the air, condensation, layered soil and vegetation, and fog above the surface.

The method which Brunt¹⁹ uses to evaluate the surface temperatures is quite general in its application and will be given here briefly. The net incoming radiation over, say, a twenty-four hour period can be represented by a Fourier Series with known coefficients (since the amount of radiation is known at least roughly). For example, if the radiation (including the absorption coefficient) varies sinusoidally during the day with maximum intensity a , and is constant at a value b during the night, a Fourier Series,

$$q_{so} = \frac{a}{\pi} - b - \frac{1}{2} a \sin \omega t - \frac{2a}{\pi} \left(\frac{1}{3} \cos 2 \omega t + \frac{1}{15} \cos 4 \omega t + \dots \right) \quad (V-2)$$

is obtained, where $\omega = \frac{\pi}{12}$.

Now if the ground surface temperature is represented by another series, with unknown coefficients,

$$T_{s,t} = T_0 + P_1 \cos (\omega t - \epsilon_1) + P_2 \cos (2 \omega t - \epsilon_2) + \dots \quad (V-3)$$

the solution, from heat-transfer work, to the temperature in a semi-infinite, homogeneous medium is

$$T_{z,t} = T_0 + P_1 \epsilon^{\mu_1 z} \cos(\omega t - \epsilon_1 + \mu_1 z) + P_2 \epsilon^{\mu_2 z} \cos(2\omega t - \epsilon_2 + \mu_2 z) + \dots \quad (V-4)$$

in which $\mu_1 = \sqrt{\frac{\omega}{2\alpha}}$, $\mu_2 = \sqrt{\frac{2\omega}{2\alpha}}$, etc.

Now, by differentiating equation V-4, putting $z = 0$ and multiplying by k , the heat flux at the surface of the soil is obtained:

$$q_{so} = k \frac{\partial T_{0,t}}{\partial z} = \sqrt{\rho c k} \sum_{n=1}^{\infty} P_n \sqrt{n\omega} \cos\left(n\omega t - \epsilon_n + \frac{\pi}{4}\right) \quad (V-5)$$

This must be identical with the flux expressed by equation V-2, and the coefficients will be obtained by comparing similar harmonic terms in each equation.

Various improvements have been suggested by Groen, since, in the above method, the net heat gain or loss by the ground is zero, and since q_0 is not really constant. The first improvement is achieved by inserting a term az in the equation V-4, which, on differentiating with respect to z and multiplying by k , becomes ka , and the second is to make q_0 in equation V-2 a function of the ground surface temperature, which then introduces unknown coefficients in both sides of the equation, leading to a little more difficulty in the calculation.

In addition to this it is possible to include the effects of the heat transferred to the air by conduction, convection, and evaporation in the following manner, which illustrates the procedure for the case of convective transfer only. The flux of heat at the ground surface is obtained by a combination of the net radiation flux, plus a term involving a surface transfer coefficient and the air-soil surface temperature difference,

$$q_{so} = q_{net} + h (T_a - T_s) \quad (V-6)$$

Now h will be a function of various factors, as described in Part III, and can be evaluated as shown therein, and the function T_a will be known. These can be approximated as coarsely as necessary and described by a Fourier Series with known coefficients, which may be combined with like terms of the radiational series. However, the soil surface temperature T involves the unknown coefficients $P_1, P_2, \dots$ as before in terms of which it must be expressed. Thus these coefficients will appear in both

expressions for q_{so} but can be evaluated, as in the above cases. When this has been carried out, the ground temperature distribution including the surface temperature becomes known, as does the heat flux.

Another, more simple and direct method has been used by Barber¹¹ for estimating the surface temperature, which involves the use of the sol-air temperature discussed in paragraph 5-02. From text books on heat transfer⁵⁴ an expression can be obtained giving the temperature distribution in a semi-infinite solid body as a function of the temperature at a point in a medium above the surface. Between the point in the medium and the surface, a surface transfer coefficient h exists. If the medium is considered to be air at a temperature

$$T_a = T_M + T_V \sin \left(\frac{\pi}{12} t \right) \quad (V-7)$$

then the temperature distribution in the soil is

$$T_{z,t} = T_M + T_V \frac{H e^{-zC}}{[(H + C)^2 + C^2]^{1/2}} \sin \left[0.262 t - zC - \tan^{-1} \left(\frac{C}{H + C} \right) \right] \quad (V-8a)$$

In this expression, if z is placed equal to 0, the surface temperature is obtained

$$T_{0,t} = T_M + T_V \frac{H}{[(H + C)^2 + C^2]^{1/2}} \sin \left[0.262 t - \tan^{-1} \left(\frac{C}{H + C} \right) \right] \quad (V-8b)$$

It remains to determine values for h , T_M , and T_V . Barber uses an equation

$$h = 1.3 + 0.62 u^{3/4} \quad (V-9)$$

given by Alford, Ryan, and Urban⁵ as holding "for forced convection including average reradiation" which presumably includes longwave outgoing radiational effects. This is an empirical expression, which cannot describe the surface transfer coefficient, except in the roughest fashion, and it should be possible to obtain a better evaluation by the methods of Part III. A discussion of average values of h is given in paragraph 5-05.

In order to determine T_M and T_V , the known radiational heat flux must be taken into account and the effective temperature will be given at any instant by equation V-1. In order to obtain the sine wave fluctuation of air temperature, Barber assumes a sinusoidal variation of radiation to give the same total daily irradiation, and this

gives a sine wave of effective temperature. That is, if the total irradiation per day is Q_{rad} langley, the average hourly contribution to effective temperature is

$$R = \frac{Q_{\text{rad}}}{24h} \quad (\text{V-10})$$

This can be roughly approximated by a sine wave of amplitude $1.5R$. If a similar sine approximation is applied to the daily temperature wave, the mean effective temperature becomes

$$T_M = T_a + R \quad (\text{V-11})$$

and the range is

$$T_V = 0.5 T_R + 1.5 R \quad (\text{V-12})$$

With a knowledge of soil properties (or pavement properties), equation V-8b can be solved. This appears to give good correlation with observed data according to Barber's figures. Barber points out that equation V-8a assumes that the temperature cycles have been prevailing for a long time and that frequently, due to a cloudy day or change in type of air mass, the mean effective temperature may rise or fall sharply from one day to the next. This means that the temperatures on the succeeding day are influenced by those of the preceding one, and will not be as calculated from equation V-8a. A further temperature change or lag, determined from equations of heat transfer,²⁵ will be superimposed on the daily variation, which can be calculated, for small values of z and large values of t , by the equation

$$T_L = \frac{z + 1/4}{\sqrt{\pi ct}} \cdot \Delta T_M \quad (\text{V-13})$$

where ΔT_M is the change in the mean value.

When the surface has been rained on, a certain amount of evaporation will take place, and this represents a heat loss to the soil which must be taken into account in the heat flux calculations. If the amount of evaporation is known, or can be estimated, the amount of heat involved can be readily included with the radiational quantity in the calculation of the effective temperature. These equations all apply to homogeneous solids, and, strictly speaking, the different properties of the soil in different layers should be accounted for. However, Barber states that numerical analysis reveals a negligible dependency of the surface temperatures on the properties below 6-in. depths.

It must be remembered that the solution to this heat-transfer problem, equation V-8a, applies to either moisture-free soils, or to temperatures altogether above or below the freezing point of soil moisture if it exists. However, for short periods of a days duration, it is unlikely that the latent heat of any ice formed during the day would significantly affect the above calculation.

In attempting to predict the surface temperature variation of a pavement over a year by temperature cycle, a correct solution would involve a two-layer system with moisture in the lower layer, subject to cyclical temperature variation and including a surface heat-transfer coefficient. Such a solution does not exist, and it is for this reason that the hydraulic analog computer was proposed. A series of solutions could be carried out on the computer demonstrating the case described above. Parameters could be given expressing the variation in surface temperature or, of course, directly yielding the depth of frost penetration in terms of atmospheric variables.

5-04. Effects of Atmospheric Variables. Although equation V-8a does not correctly describe the phenomena concerned in the freezing of layered systems, it is considered worth while to assume reasonable values of the various coefficients and parameters involved in order to plot the different surface temperatures obtained by varying different terms in the equation.

Tables V-1 and V-2 contain values of pavement surface temperature for both concrete and asphalt surfaces under varying amounts of incoming shortwave radiation. A fixed quantity is assigned to longwave outgoing radiation, and values are given for

Table V-1
Temperature Fluctuation at Ground Surface - Variation
of Incoming Radiation

Bituminous Concrete ($F = 0.725$)					
R	200	300	400	500	600
T_M	62.77	65.69	68.62	71.56	74.47
$F \cdot T_V$	13.9	17.07	20.25	23.45	26.63
54-min lag					
Concrete ($F = 0.700$)					
R	200	300	400	500	600
T_M	60.92	62.92	64.92	66.92	68.92
$F \cdot T_V$	11.48	13.58	15.62	17.78	19.87
61-min lag					

Table V-2

Temperature Fluctuation at Ground Surface - Variation
of Surface Transfer Coefficient

Incoming Radiation = 500 cal/cm²/day

Outgoing Radiation = 100 cal/cm²/day

Bituminous Concrete*					
h	3	4	5	6	8
F	0.604	0.674	0.725	0.763	0.815
T _M	79.24	74.43	71.56	69.61	67.21
F · T _V	26.5	24.7	23.45	22.45	21.10
*Bituminous concrete absorbs 375 cal/cm ² /day					
Concrete**					
h	3	4	5	6	8
F	0.574	0.646	0.700	0.740	0.795
T _M	71.54	68.65	66.92	65.77	64.32
F · T _V	18.57	18.08	17.78	17.54	17.12

**Concrete absorbs 225 cal/cm²/day

cases of fixed incoming radiation, and for different magnitudes of transfer coefficient. The values from these tables have been plotted in Figures V-1 and V-2, which demonstrate the diurnal variation in surface temperature under the assumed conditions. The importance of the absorptivity of the surface is seen since this quantity has a major effect on the heat economy of the ground. The curves all have a time lag which depends on the heat properties of the soil and the air only; thus the lag is the same for all the curves for concrete in Figure V-1 and has a different value for asphalt. In Figure V-2, however, the amount of lag depends on the value of h and the dependence of temperature is well shown.

It will be observed that in equation V-8a no allowance is made for the specific heat of the air, which is considered negligible, and that the variation in solar intensity and air temperature are held to be contemporaneous. The way in which nighttime outgoing longwave radiation effects are considered is also artificial, and the nighttime soil temperatures must not be supposed to be at all close to nature, although the daytime temperatures may give an adequate representation. In addition, the values of incoming solar radiation are in each case considered to vary sinusoidally over the day; on a cloudy day, corresponding to the 200-300 langley/day values, this will not be true.

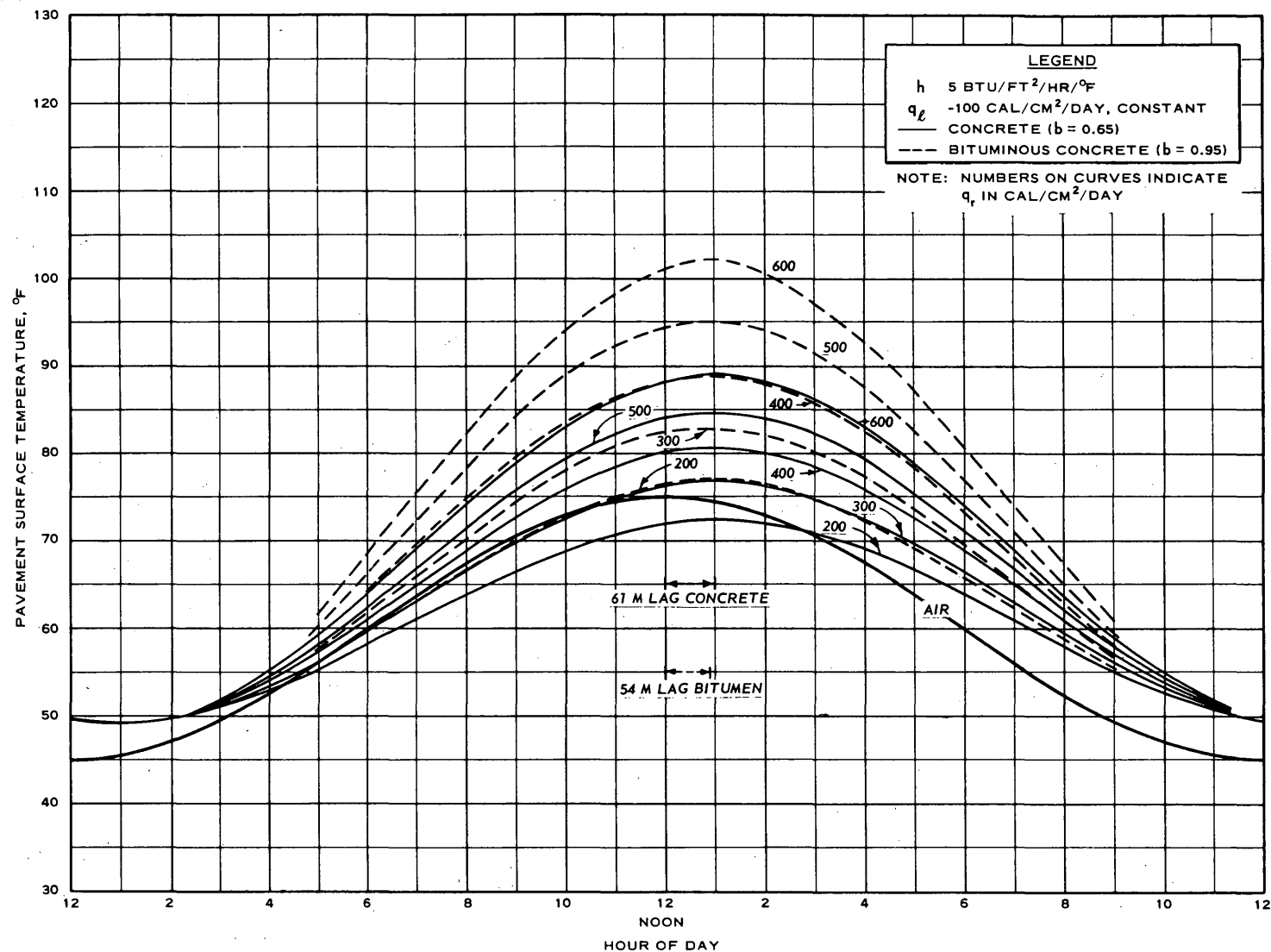


Figure V-1. Pavement surface temperature vs time and incoming solar radiation

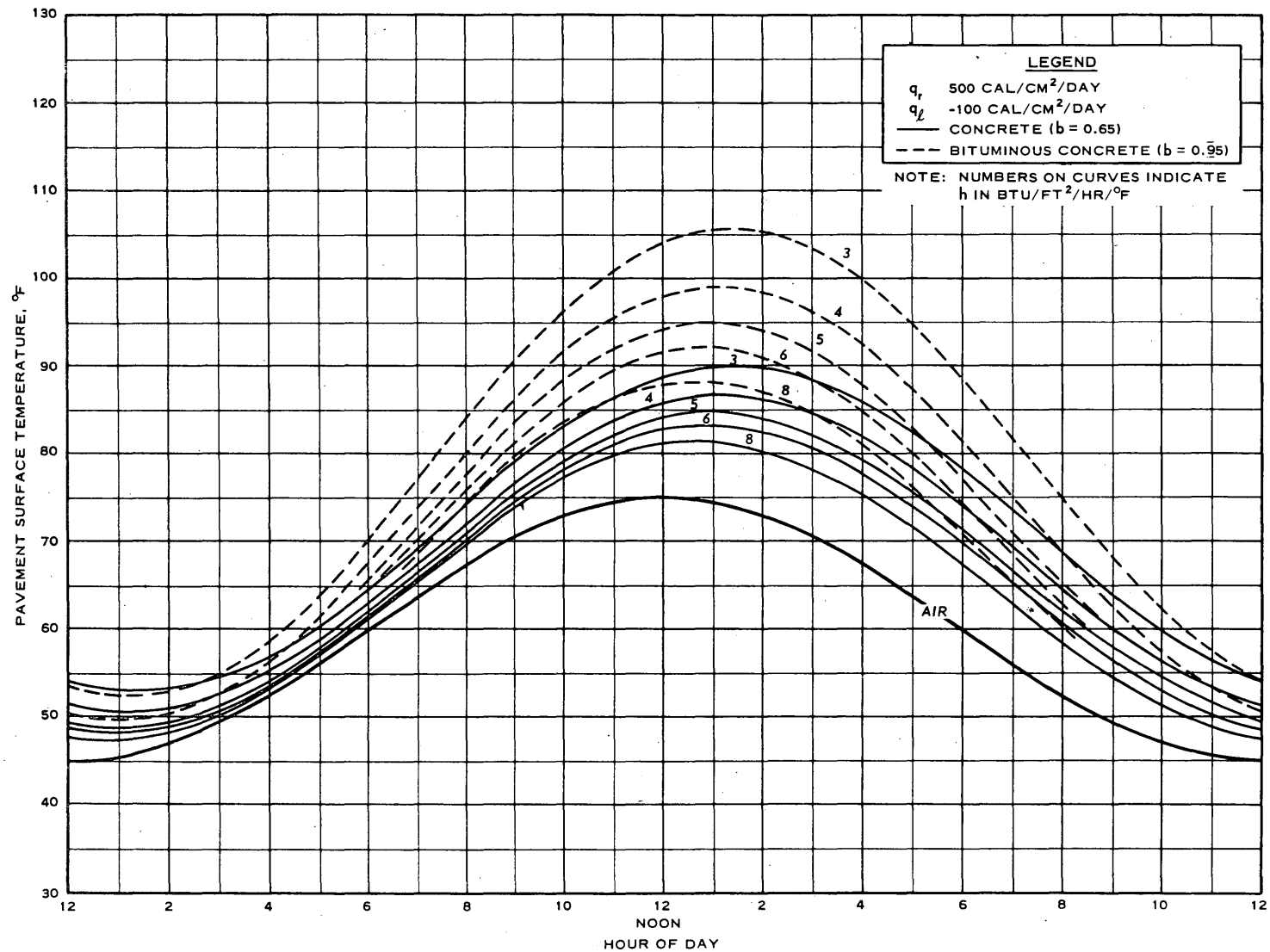


Figure V-2. Pavement surface temperature vs time and surface transfer coefficient

5-05. Average Value of Surface Heat-Transfer Coefficient. The problem of altering the computer component corresponding to the air-soil heat-transfer coefficient to simulate the variations in the coefficient is a difficult one, and, although it might be possible to mechanize the resistor to perform according to a program, a better solution would be to find some average value at which it could be set to conform to the natural problem. In many cases, the variation in magnitude of the surface transfer coefficient is very large over a period of even a few hours, and its connection with the actual temperature gradient in the air complicates analyses considerably. That is to say, a large temperature gradient leads usually to a large transfer coefficient (and also winds are generally stronger in the middle of the day) and to the movement of large quantities of heat to the air from the ground surface. On the other hand, at night, when the cooler ground leads to cooler air layers near the ground and to a possible inversion, or at least to positive damping effects, the transfer coefficient is small (as are also usually the temperature gradients) and only small amounts of heat transfer take place.

The determination of an average value of the coefficient, when initially no value of ground surface temperature is known, becomes a difficult task. Another question is raised, which is to decide what effect the average value has to have on the temperature regime. If the whole process of heat transfer to the air is concerned, in which, in the course of time, the temperature of the air at some elevation above the ground surface fluctuates, and in which the heat-transfer coefficient also fluctuates, then the fluctuation of the ground surface temperature depends on both the above variations, and no average value of one or the other will give the same ground surface temperature-time curve. In the case where freezing or thawing is considered, the important quantity will be the amount of heat transferred between the ground and the air. This amount determines how much freezing or thawing takes place, although the rate will not be correct, nor will the time at which the process concerned reaches a particular depth be the same as in the variable case. Since the flux of heat into or out of the ground is the result of the differences between the other heat terms in the budget, it follows that the average heat-transfer coefficient which gives the same net heat flux to the air over the period involved will satisfy the immediate conditions.

Consequently the problem becomes one of determining a value for the surface heat-transfer coefficient, which, used in conjunction with the fixed temperature curve (which may be either actual or effective temperature depending on the method used to take into account the radiation terms of the process), will give, over a fixed period, the correct amount of heat flow to the air. Over an average annual cycle, of course,

the net heat flux into the ground is zero, and this will also be true for the daily cycle at certain times of the year, although the air heat flow will not be zero over the same period except in unusual circumstances.

Naturally, a general solution is first hoped for, but on closer inspection this proves to be very difficult indeed, if not impossible. If the problem is approached mathematically, it is necessary to utilize the method, outlined in paragraph 5-03, of assigning a Fourier Series to the ground surface temperature variation.

The heat flux which results from the solution, with its unknown coefficients, is compared with the known heat flux from the air temperature and radiation data, together with an assumed variation of the surface transfer coefficient. Thus the unknown coefficients and phase lag terms are found; when they are known, the heat flux to the air over any period of the time considered can be calculated. The problem then becomes one of finding an average value of the transfer coefficient over that period to give the same total flux.

It was decided to investigate the solution of a problem involving a diurnal variation of the heat budget in which no solar radiation was considered and the only heat flow was to the air. The temperature of the air was assumed to vary sinusoidally, reaching a maximum at midday, and having the form

$$T_a - T_M = T_V \sin \omega t \quad (V-14)$$

The transfer coefficient had a similar variation, being a maximum also at midday due to the turbulence and generally higher winds, giving

$$h - h_M = h_V \sin \omega t \quad (V-15)$$

The ground surface temperature is

$$T_s - T_o = P_1 \cos (\omega t - \epsilon_1) + P_2 \cos (\omega t - \epsilon_2) + \dots \quad (V-16)$$

and, from the generalized solution, assuming a variation of mean temperature with depth, the heat flux is obtained at the ground surface

$$\begin{aligned}
q_{so} = & ka + \sqrt{\rho ck \omega} \cdot P_1 \cos \left(\omega t - \epsilon_1 + \frac{\pi}{4} \right) \\
& + \sqrt{\rho ck \cdot 2\omega} \cdot P_2 \cos \left(\omega t - \epsilon_2 + \frac{\pi}{4} \right) \\
& + \dots \dots \dots
\end{aligned}
\tag{V-17}$$

Now the heat flux to the air, from equations V-14, -15, and -16, is

$$\begin{aligned}
q_{so} = & (h_M + h_V \sin \omega t) \left[(T_M - T_O) + T_V \sin \omega t \right. \\
& \left. - P_1 \cos (\omega t - \epsilon_1) - P_2 \cos (2 \omega t - \epsilon_2) + \dots \right]
\end{aligned}
\tag{V-18}$$

Equation V-18 can be developed and the coefficients of the constant, $\sin \omega t$, $\sin 2 \omega t$, ..., $\cos \omega t$, $\cos 2 \omega t$, ... terms can be evaluated. When they are compared to the similar terms in equation V-17, the following equations result.

Constant coefficient:

$$h_M (T_a - T_O) + \frac{T_V h_V}{2} - h_V P_1 \frac{\sin \epsilon_1}{2} = ka$$

Sin qt coefficient:

$$T_V h_M + h_V (T_M - T_O) - h_M P_1 \sin \epsilon_1 + h_V P_2 \frac{\cos \epsilon_2}{2} = \sqrt{\rho ck \omega} \cdot P_1 \sin \left(\epsilon_1 + \frac{\pi}{4} \right)$$

Sin 2 qt coefficient:

$$-h_M P_2 \sin \epsilon_2 + h_V P_3 \frac{\cos \epsilon_3}{2} = \sqrt{\rho ck \cdot 2\omega} P_2 \cdot \sin \left(\epsilon_2 + \frac{\pi}{4} \right) \text{ etc.}$$

Cos qt coefficient:

$$-h_M P_1 \cos \epsilon_1 - h_V P_2 \frac{\sin \epsilon_2}{2} = \sqrt{\rho ck \omega} \cdot P_1 \cdot \cos \left(\epsilon_1 + \frac{\pi}{4} \right)$$

Cos 2 qt coefficient:

$$-\frac{T_V h_V}{2} - h_M P_2 \cos \epsilon_2 + h_V P_1 \frac{\sin \epsilon_1}{2} - h_V P_3 \frac{\sin \epsilon_3}{4} = \sqrt{\rho ck \cdot 2\omega} \cdot P_2 \cos \left(\epsilon_2 + \frac{\pi}{4} \right) \text{ etc.}$$

The identities can be carried on to express all the coefficients desirable, but in practice no easy evaluation is possible unless the number is severely limited to, say, P_1 , ϵ_1 and P_2 , ϵ_2 . Then, in a practical case, the substitutions of appropriate constants can be made, and the above identities reduce to trigonometric equations. It is seen that there is little possibility of solving the above case in a simple form, finding

the heat flux, and finally the average value of transfer coefficient to give this heat flux. The problem is complex because the ground surface temperature has first to be determined; if a problem was given, including the surface temperature, a solution would be much easier.

As has been mentioned, ground surface temperature data are given in some of the Johns Hopkins Seabrook Farms publications,⁵⁵ and it is possible to use this information to calculate, for one case, an average value of transfer coefficient, to compare it with the magnitude of daily fluctuations, and make certain deductions from the result.

The date of August 21, 1952, has been chosen as suitable, since the ground surface temperature is given throughout an entire twenty-four hour period; Table V-3 shows the appropriate results. Figure V-3 presents curves of temperature difference between the ground and a height of 160 cm, average values of h , and heat flux to the air, for each hour of the twenty-four. The values of h have been calculated by the methods outlined in Part III. It will be seen from Figure V-3 that, on this particular summer day in a temperate climate, most of the heat flux to the air takes place during the day outward from the soil surface and that only a very small amount of heat is transferred to the soil at night. In addition, while the amount of heat transferred depends both on the large temperature differences and the large transfer coefficients prevailing during daylight hours, the temperature differences determine the amount more significantly. Thus, in order to arrive at an average value of heat-transfer coefficient in this case, it would appear in order to neglect entirely the nighttime processes and to work on the basis of the temperatures and transfer coefficients prevailing in the daylight hours.

Thus, in this case, the average daylight temperature difference is 2.06°C . between the air at 160 cm and the ground surface, and the average daylight transfer coefficient is $1.51\text{ cal/cm}^2\text{/hour/}^{\circ}\text{C}$. which, combined, give $3.11\text{ cal/cm}^2\text{/hour}$ or about 31 cal. for the daylight period, which is very close to the net heat transferred to the air over the whole twenty-four hour period. This would suggest that in such circumstances, for long-period operation of the analog computer, information on the average daylight air temperature and on the daylight wind conditions averaged over the twenty-four hour period would yield the requisite data for programming the computer with fair accuracy. As a matter of interest, the average day and night heat-transfer coefficient is $0.99\text{ cal/cm}^2\text{/hour}$, and the average temperature difference is -0.62°C ., which gives rise to a total amount of heat transferred from the

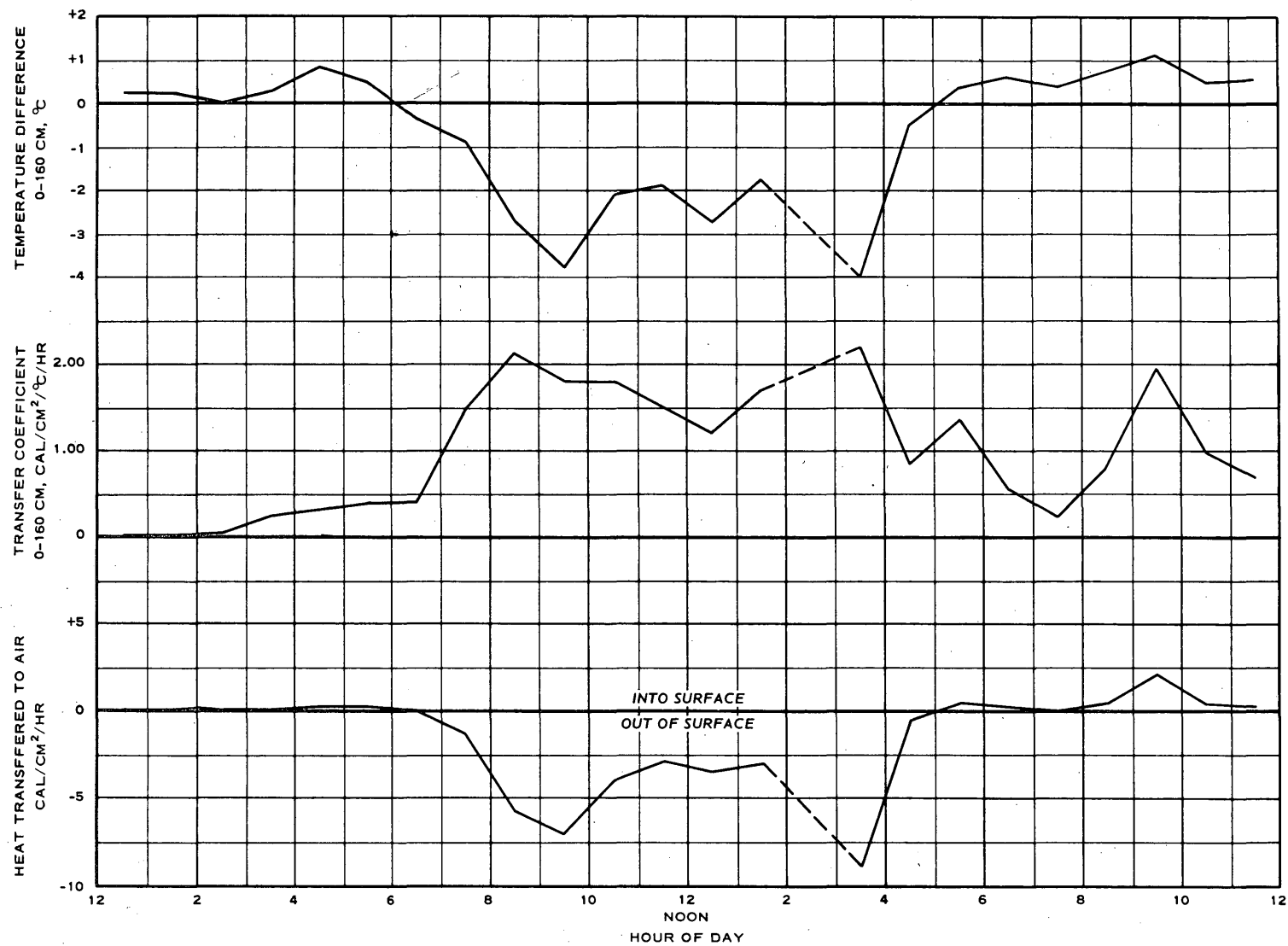


Figure V-3. Heat transfer to air, 21 August 1952, Seabrook, N.J.

Table V-3

Heat Transferred to Air
21 August 1952, Seabrook Farms, N. J.

Time	T_{O-160}	K_{H40-80}	K_{HO-160}	$h \times 10^{-2}$	q_{co}
00-01	+0.23	7.82	2.99	1.93	+0.00444
01-02	+0.20	4.73	1.80	1.17	+0.00234
02-03	-0.02	15.24	5.8	3.77	-0.00075
03-04	+0.25	97.4	37.0	24.1	+0.0602
04-05	+0.84	123.4	47	30.6	+0.257
05-06	+0.51	158.3	60	39.0	+0.199
06-07	-0.37	277	61	39.7	-0.147
07-08	-0.89	1037	228	148.4	-1.323
08-09	-2.66	1484	326	212	-5.64
09-10	-3.76	1266	278	181	-6.80
10-11	-2.12	1257	277	180	-3.81
11-12	-1.89	1046	230	150	-2.83
12-13	-2.73	857	188	122	-3.33
13-14	-1.73	1196	263	171	-2.96
14-15					
15-16	-3.97	1538	338	220	-8.73
16-17	-0.45	585	129	83.9	-0.378
17-18	+0.39	939	207	135	+0.526
18-19	+0.62	229	87	56.5	+0.350
19-20	+0.41	102	39	25.4	+0.104
20-21	+0.74	325	124	80.7	+0.597
21-22	+1.12	791	301	196	+2.19
22-23	+0.47	392	149	97	+0.456
23-24	+0.53	280	107	69.6	+0.369

ground to the air of $14.7 \text{ cal/cm}^2/\text{day}$, which is much lower than the previously calculated amounts. Obviously, too much weight is given by such a method to the very low transfer coefficients and gradients operating at night, and thus average daily temperature and wind velocity data are misleading.

Unfortunately, no data are available on surface temperatures at night in the winter periods, when any convective processes will assume a greater importance relative to the radiational heat losses. In long arctic nights the situation will exist

where average wind velocities and temperatures for the whole twenty-four hour period will suffice to obtain solutions. In areas and at times of the year where the daylight hours are short it is difficult to suggest methods without analysis of further data obtained in such situations. If the maximum and minimum temperatures are given for each day, then, assuming a sinusoidal variation, it is possible to obtain an idea of the average daylight and average nighttime temperature to utilize in conjunction with wind data. It is usual, however, only to use average wind velocities, in which case little information of value can be obtained. It would seem that each problem should be considered against its own particular background of latitude and time of year.

5-06. Meteorological Data Needed. It becomes obvious from the preceding discussion that meteorological information of a specialized nature is desired for a complete analysis of air-soil heat transfer, and recommendations of the form this information should take are included herewith as a guide to future studies. Certain assumptions must be borne in mind when examining the proposals, and these will be briefly mentioned. The chief aim of analyses at present is to predict subsurface temperatures below airfield pavements, and consequently rainfall and evaporation data are relevant only to the extent that the pavement will be wetted by a certain minimum rainfall until a small amount of water accumulates on its surface; additional rainfall will run off. Thus, the frequency and distribution of rain are of more interest than the intensity or amount of water precipitated. The information desired may or may not be complete; much depends on the results of investigations which should be carried out on temperature and wind profile measurements taken in different latitude zones.

5-07. Rainfall. It would be convenient to have tabulations of the number of independent rainfalls per week during the thawing and refreezing season plus two weeks on each side.

5-08. Temperature. The desirable temperature records would be:

a. Mean weekly daylight temperature throughout the year at 4 ft, or at some other definitely specified height, above ground surface. The temperature referred to is the daylight one, as it has been found so far that the quantity of heat transferred by the air to the ground at night is probably not significant.

b. Mean weekly daylight temperature for coldest winter in record (or in 5 or 10 years).

c. Mean weekly daylight temperature for warmest summer in record (or in 5 or 10 years).

5-09. Humidity. A satisfactory humidity record would consist of mean monthly vapor pressure throughout the year. This is required for solar radiation calculations, and is therefore desirable as an estimate of atmospheric humidity.

5-10. Wind. A record is desired containing mean weekly daylight wind velocities throughout the year at 4 ft, for the same reasons given in paragraph 5-08a.

5-11. Sunshine.

a. Desirable records of sunshine would include:

- (1) Mean weekly total hours of bright sunshine.
- (2) Mean weekly minimum total hours of bright sunshine.
- (3) Mean weekly maximum total hours of bright sunshine.

Alternatively, in this case, it is necessary to have information about cloud cover, but it is probably difficult both to ascertain and express these data adequately.

b. In order to check calculations of solar radiation received weekly on a horizontal surface from estimates of weekly total hours of bright sunshine and humidity, it would also be useful to receive information regarding any direct radiation measurements near the sunshine data stations.

c. For nighttime radiation purposes, some estimate of average weekly cloud cover at night is desirable, if such information can be obtained.

5-12. Approaches to Utilization of Data. In this part of the report, the broad problem of applying the information obtained in previous analyses to the operation of the computer has been considered. Two approaches present themselves: the more obvious one, the possibility of utilizing the data in the form in which they are received and calculated to run the analog which will then give the required ground surface and sub-soil temperatures; and the other, the possibility of using the information to calculate the surface temperature by various analytical, semiempirical methods, which temperature can then be programmed into the computer.

5-13. First Approach. The first approach can be subdivided into two techniques for the more convenient handling of the variables, and these are both discussed. In both, the radiational heat flux is transformed into an equivalent air temperature, which can be easily represented on the apparatus to operate across an appropriate surface transfer coefficient. The difference between the two methods lies in the handling of the transfer coefficient. If complete information on the wind and temperature data is available, the variation of the transfer coefficient can be taken into account at all times, and this can be applied to the computer by continuously varying the resistance of the element representing the transfer coefficient.

As has been pointed out, however, it is not easy to achieve this adjustment, and it is desirable to utilize a single resistance setting, if possible. By means of an analysis of some heat-transfer data it has been shown possible to find an average value to describe the process. If the variation of the temperature and wind velocity data is known continuously throughout the period of observation, this average value can be fairly readily computed and it appears from the calculations that, for noncontinuous data, an average based on the daytime average wind velocity would give reasonably good results, rather than one based on the over-all daily average, since only very small amounts of heat transfer to and from the air take place at night.

5-14. Second Approach. The second approach is unfortunately difficult to carry through mathematically and will have to be investigated more thoroughly for particular cases. However, it appears that daily surface temperatures can be fairly adequately duplicated, and a possibility exists of calculating the average surface temperatures over longer periods, providing a simple, describable fluctuation of all the factors involved is assumed.

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APPENDIX A: LIST OF SYMBOLS

Symbol	Part or Equation*	Meaning	Dimension**
a	III-9	Constant	-
a	III-27	Temperature gradient coefficient	θ/L
a	V-2	Intensity of radiation	M/T^3
b	IV-10	Constant	-
b	V-1	Absorptivity of a surface	-
b	V-2	Intensity of radiation	M/T^3
c	III-2	Volumetric specific heat	$M/T^2 L \theta$
c	IV-14	Constant	-
c	V-5	Gravimetric specific heat	$M/T^2 L \theta$
d	IV-14	Constant	-
e	III-10	Base of natural logarithms	-
e	IV-14	Vapor pressure	M/LT^2
e_o, e_z	III-20	Vapor pressure at point	M/LT^2
$e_{1,2,\dots}$	III-16	Partial water vapor pressure at points 1,2,...	M/LT^2
f	IV-14	Constant	-
g	III-7	Acceleration of gravity	L/T^2

*First usage

**M, Mass; L, length; T, time; θ , temperature.

Symbol	Part or Equation	Meaning	Dimension
g	IV-14	Constant	-
h	V-1	Surface heat-transfer coefficient	$M/T^3\theta$
h_M	V-15	Mean value of h	$M/T^3\theta$
h_V	V-15	One-half amplitude of h variation	$M/T^3\theta$
h_O	III	Height of source surface above ground level	L
k	V-5	Thermal conductivity	$M/T^3\theta$
k_O	III-4	von Karman constant (0.4)	-
l	III-3	Mixing length	L
m	III-27	Exponent coefficient	1/L
n	III-8	Number indicating degree of turbulence	-
n	V-5	Number	-
p	IV-1	Transmission coefficient	-
p	IV-6	Atmospheric pressure	M/LT^2
p^m	IV-2	Function of wavelength of radiation	-
q	III	Heat flux	M/T^3
q	III-10	Specific humidity of atmosphere	M/M
q_e	III-16	Evaporative heat loss	M/T^3
q_{ro}	III-1	Heat flux at ground surface due to shortwave radiation	M/T^3

Symbol	Part or Equation	Meaning	Dimension
q_{lo}	III-1	Heat flux at ground surface due to longwave radiation	M/T^3
q_{co}	III-1	Heat flux at ground surface due to convection in the air	M/T^3
q_{eo}	III-1	Heat flux at ground surface due to evaporation	M/T^3
q_{so}	III-1	Heat flux at ground surface due to conduction in the soil	M/T^3
q_{rc}	IV-10	Shortwave radiational heat flux on a cloudy day	M/T^3
q_{rs}	IV-10	Shortwave radiational heat flux on a clear day	M/T^3
q_{le}	IV-13	Longwave radiational heat flux from earth's surface	M/T^3
q_{la}	IV-14	Longwave radiational heat flux from atmosphere	M/T^3
q_{lc}	IV-16	Net longwave radiational heat flux with clouds	M/T^3
t	III-2	Time	T
t	IV-4	Hour angle of the sun	-
u, v, w	III-3	Total velocity components in x, y, z directions; $u = \bar{u} + u'$, etc.	L/T
$\bar{u}, \bar{v}, \bar{w}$	III-3	Mean velocity components in x, y, z directions	L/T
u', v', w'	III-3	Magnitude of velocity fluctuation from mean velocity	L/T

Symbol	Part or Equation	Meaning	Dimension
$u_{1,2, \text{etc.}}$ $v_{1,2}$	III-3	Total velocity components at points 1, 2, etc.	L/T
u_*	III-5	Friction velocity = $(\tau_o/\rho)^{1/2}$	L/T
w	IV-6	Amount of precipitable water in a layer of the atmosphere	L
w_t	IV	Total amount of precipitable water in the atmosphere above a point	L
x, y, z		Distance, height	L
$x_{1,2, \dots}$ $y_{1,2, \dots}$		Distance, height, to points 1, 2, ... etc.	L
z_o	III-4	Roughness dimension	L
A	III-16	Constant	-
B	III-17	Constant	1/L
C	III-20	Constant	1/L
C	IV-11	Cloud cover in tenths	-
C	V-8	Factor = $(\pi/24\alpha)^{1/2}$	1/L
C_n	III-28	One-half amplitude of temperature wave at depth z_n	θ
D	III-19	Molecular vapor diffusivity	L^2/T
D	IV-4	Sun's declination	-
E	III-10	Flux of water vapor	M/L^2T

Symbol	Part or Equation	Meaning	Dimension
F	Table V-1	Factor = $\frac{H}{[(H + C)^2 + C^2]^{1/2}}$	-
F _{1, 2, ...}	III-32	Proportionality constants	L/T
H	V-8	Factor = h/k	-
I	IV-1	Heat flux due to solar radiation after depletion	M/T ³
I _O	IV-1	Heat flux due to solar radiation outside atmosphere normal to a surface (1.94 cal/cm ² /min)	M/T ³
K	III-15	Eddy diffusion coefficient	L ² /T
K _{M, V, H}	III-3	Eddy diffusion coefficient of momentum (mass), vapor, or heat	L ² /T
K _{H1, 2, ...}	III-25	K _H at points 1, 2, ... etc.	L ² /T
K _{Ht}	III-30	K _H over range including both molecular and turbulent diffusion	L ² /T
L	IV-4	Latitude	-
P _{1, 2, ...}	III-26	Fourier Series coefficients	θ
R	III-1	Bowen ratio	-
R	III	Gas constant for dry air (2.876 x 10 ⁶ cm ² /sec ² /°C.)	L ² /T ² θ
Ri	III-7	Richardson number	-
S	IV-10	Percentage of sunshine	-
T	III-2	Temperature	θ

Symbol	Part or Equation	Meaning	Dimension
$\bar{T}$	III-7	Temperature gradient	θ
$T_{s,t}$		Temperature at surface at time t	θ
T_a	IV-14	Air temperature	θ
T_s	III-30	Surface temperature	θ
T_E	V-1	Effective air temperature	θ
T_M	III-26	Mean air temperature	θ
T_V	V-7	One-half amplitude of temperature variation	θ
T_L	V-13	Temperature lag	θ
$T_{2,3,\dots}$	III-34	Temperature at points 2,3 etc.	θ
Z	IV-2	Zenith angle, degrees or radians	-
α	V-4	Thermal diffusivity $k/\rho c$	L^2/T
$\alpha_{x,y,z}$		Thermal diffusivity in x,y,z directions	L^2/T
β	III-9	Exponent representing degree of stability	-
γ	IV-14	Constant	-
$\delta_{H,M}$	III-31	Thickness of thermal, momentum boundary layer	L
ϵ	III	Equivalent sand roughness	L

Symbol	Part or Equation	Meaning	Dimension
$\epsilon_{1,2}$	III-26	Lag terms in radians	-
e	IV-14	Base of natural logarithms	-
$\mu_{1,2\dots}$	V-4	Exponent factor = $(\omega/2\alpha)^{1/2}, (2\omega/2\alpha)^{1/2}, \dots$	1/L
ν	III-5	Kinematic viscosity = μ / ρ	L^2/T
ρ	III-3	Air density	M/L^3
ρ	V-5	Soil density	M/L^3
σ	IV-13	Stefan-Boltzmann constant $(8.26 \times 10^{-11} \text{ cal/cm}^2 \text{ min/}^\circ\text{C.}^4)$	$M/T^3 \theta^4$
τ	III-3	Shear stress	M/LT^2
ω	V-2	Frequency	1/T
Γ	III-7	Adiabatic lapse rate (1°C./100 m)	θ/L
δ	III-19	Thickness of laminar sublayer	L

Most of the conversion factors from the metric to the English system of units are well known, but some convenient ratios are given below. The gravimetric specific heats in the metric and English systems require, of course, a conversion factor of unity by definition.

Unit	Metric	Conversion	English
Thermal conductivity	$\text{cal/cm}^2/^\circ\text{C./min}$	123.0	$\text{BTU/ft}^2 \text{ }^\circ\text{F. hr}$
Thermal diffusivity	cm^2/min	0.0645	ft^2/hr
Volumetric specific heat	$\text{cal/cm}^3/^\circ\text{C.}$	62.4	$\text{BTU/ft}^3 \text{ }^\circ\text{F.}$
Heat flux	$\text{cal/cm}^2/\text{min}$	221.4	$\text{BTU/ft}^2 \text{ hr}$
Density	gm/cm^3	62.4	lb/ft^3